Los Alamos NATIONAL LABORATORY

$H \neq H^{\dagger}$

Non-Hermitian Quantum Mechanics: Physical Implications

Avadh Saxena

Collaborators: Julia Cen (LANL), G.W. Chern (UVa), Y.N. Joglekar (IUPUI)

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avadh@lanl.gov

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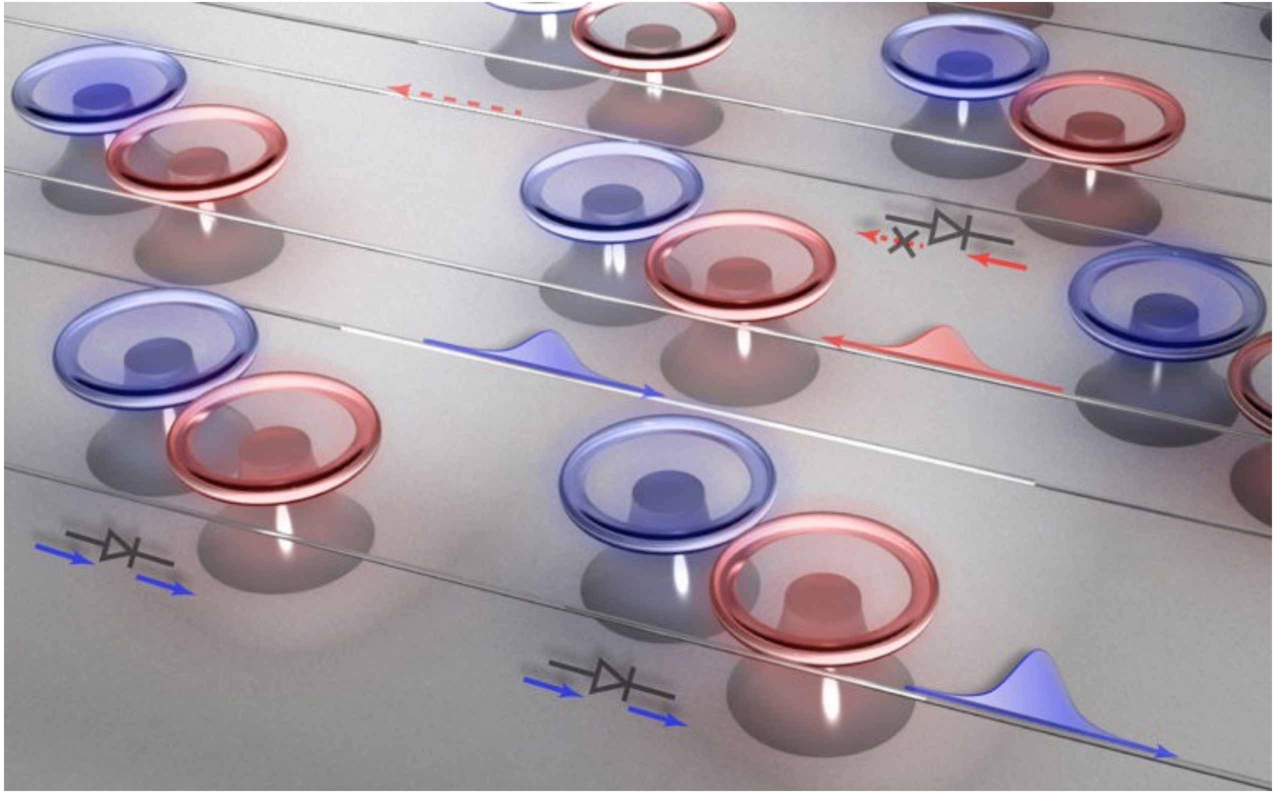
OUTLINE:

- Extended Introduction: $\mathcal{P}$**arity-**$\mathcal{T}$**ime** and **anti-**$\mathcal{PT}$-symmetry
- $\mathcal{PT}$**-** and **anti-**$\mathcal{PT}$-symmetric qubits
- Discrete case: $\mathcal{PT}$-symmetric Kagome lattice
- Applications: Double-Drive Floquet $\mathcal{PT}$-system
- Quantum sensing: Next talk (A. Harter)
- Conclusion: non-Hermitian qubits better, useful.

Experiment: Coupled Optical Resonators

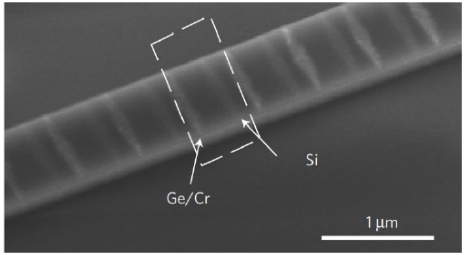


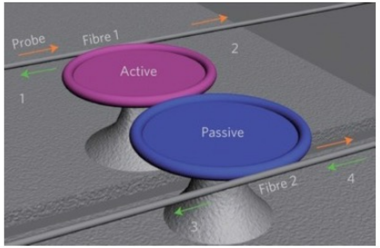
Light can only travel one way (instead of both directions): PT-symmetric red/blue toroids

Recent Developments:

Reflectionless $\mathcal{PT}$ metamaterial

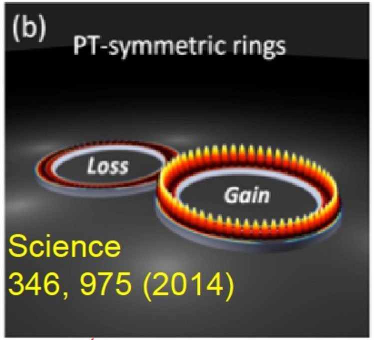
Nature Mat. 12, 108 (2013)



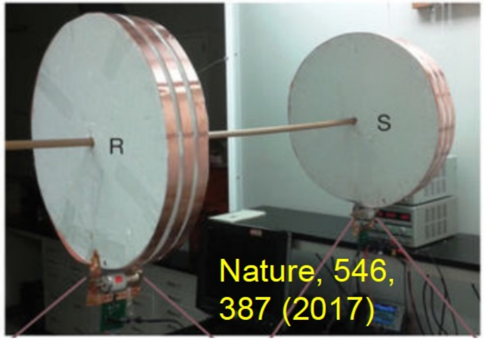


$\mathcal{PT}$ –isolators

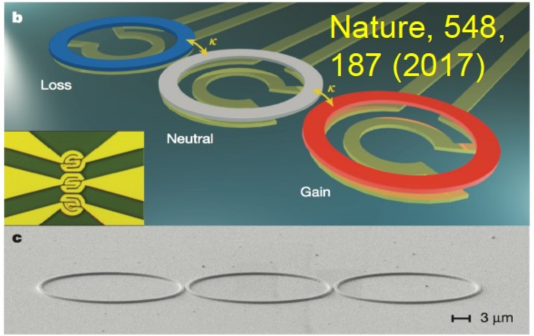
Nature Phot. 8, 524 (2014)



$\mathcal{PT}$- nanolaser



Wireless power transfer



Realization of EP3

$\mathcal{PT}$-symmetric quantum systems

Bender & Boettcher, PRL **80**, 5243 (1998)

$\mathcal{P}$arity-$\mathcal{T}$ime symmetry:

$$\mathcal{P}\,x\,\mathcal{P} = -x \quad \text{and} \quad \mathcal{P}\,p\,\mathcal{P} = -p$$
$$\mathcal{T}\,x\,\mathcal{T} = x, \quad \mathcal{T}\,p\,\mathcal{T} = -p \quad \text{and} \quad \mathcal{T}\,i\,\mathcal{T} = -i$$

$\mathcal{PT}$-symmetric Hamiltonian: $[\mathcal{PT}, H] = 0$

- unbroken regime: all eigenvalues real
- broken regime: complex and real eigenvalues

Definition of the inner product:

$$\langle\psi_1|\psi_2\rangle_{\mathcal{CPT}} = (\mathcal{CPT}\psi_1)\cdot\psi_2$$

Metric operator (unbroken regime):

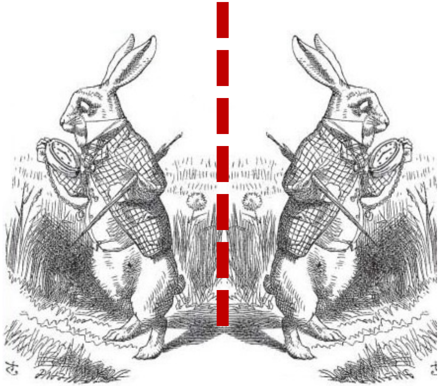
$$[\mathcal{C}, H] = 0 \quad \text{and} \quad \mathcal{C}^2 = \mathbb{I}$$

$\mathcal{PT}$-symmetry

Bender & Boettcher, PRL 1998

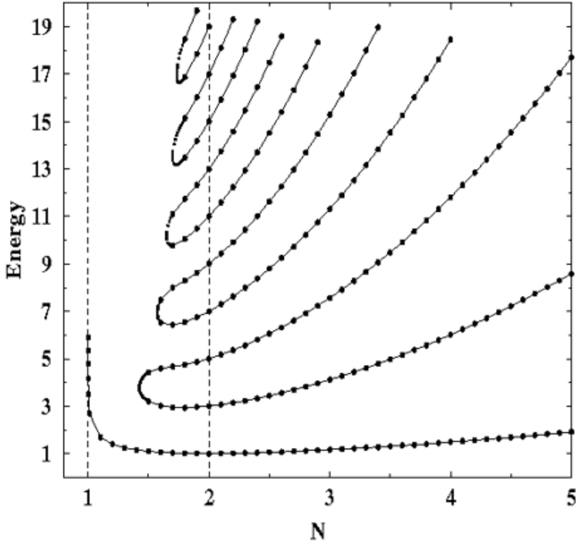
$\mathcal{P}: x \to -x, \quad p \to -p$

$\mathcal{T}: x \to x, \quad p \to -p, \quad i \to -i$



$[H, \mathcal{PT}] = 0$

$\mathrm{H} = p^2 - (ix)^N$



Review Paper (expt.)

nature physics

REVIEW ARTICLES

PUBLISHED ONLINE: 5 JANUARY 2017 | DOI: 10.1038/NPHYS4323

Non-Hermitian physics and PT symmetry

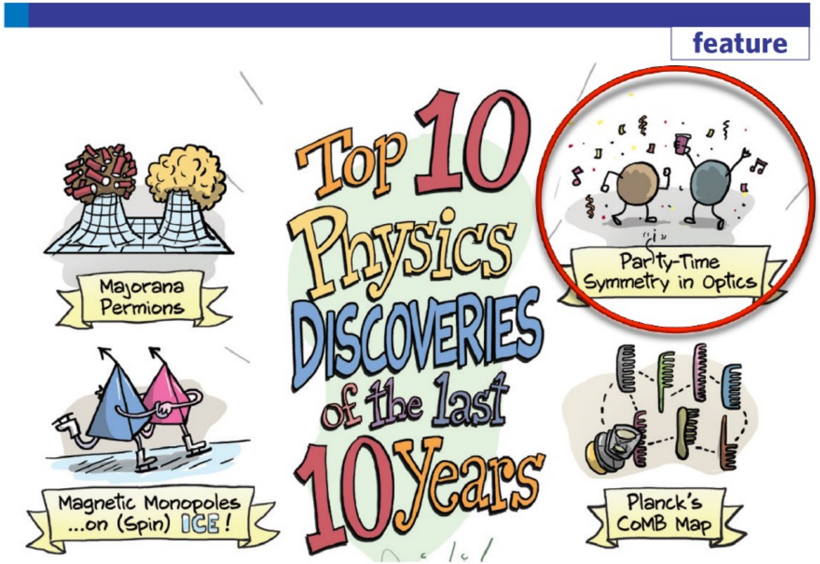
Ramy El-Ganainy[1], Konstantinos G. Makris[2], Mercedeh Khajavikhan[3], Ziad H. Musslimani[4], Stefan Rotter[5] and Demetrios N. Christodoulides[3]*

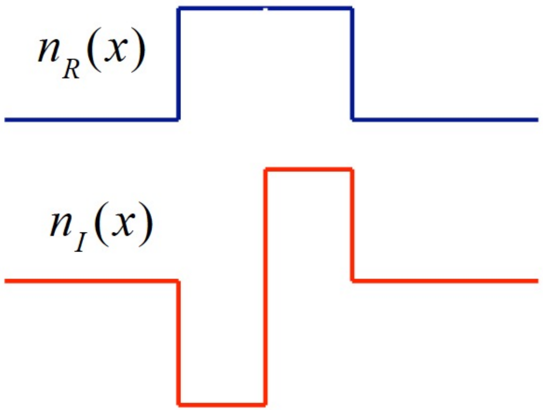
Parity-Time Symmetry in Optics

Should a Hamiltonian be Hermitian in order to have real eigenvalues?

C. M.Bender, S. Boettcher, Phys. Rev. Lett., 80, 5243 (1998)

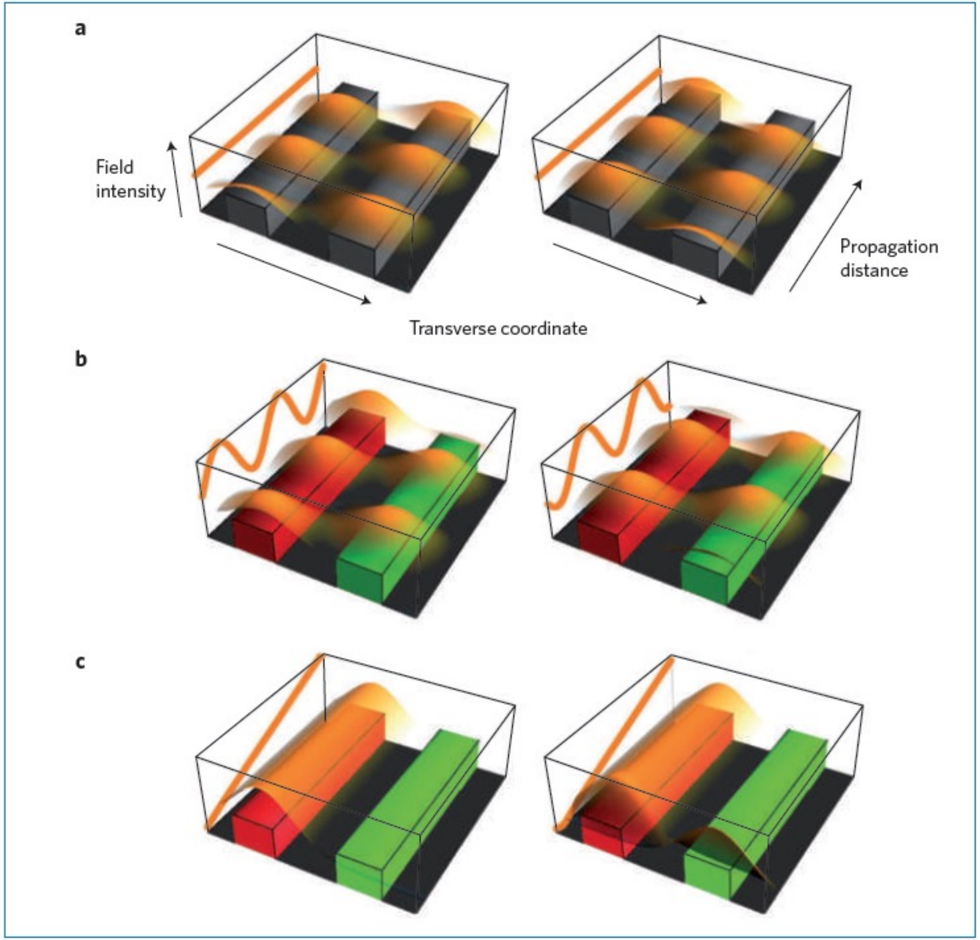
$$n(\vec{r}) = n^{*}(-\vec{r})$$





K. G. Makris, R. El-Ganainy, D. N. Christodoulides, and Z. H. Musslimani, Phys. Rev. Lett. 100, 103904 (2008)
C. Ruter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev and D. Kip, Nat. Phys. 6, 192 (2010)

LIGHT PROPAGATION IN A PT MATERIAL



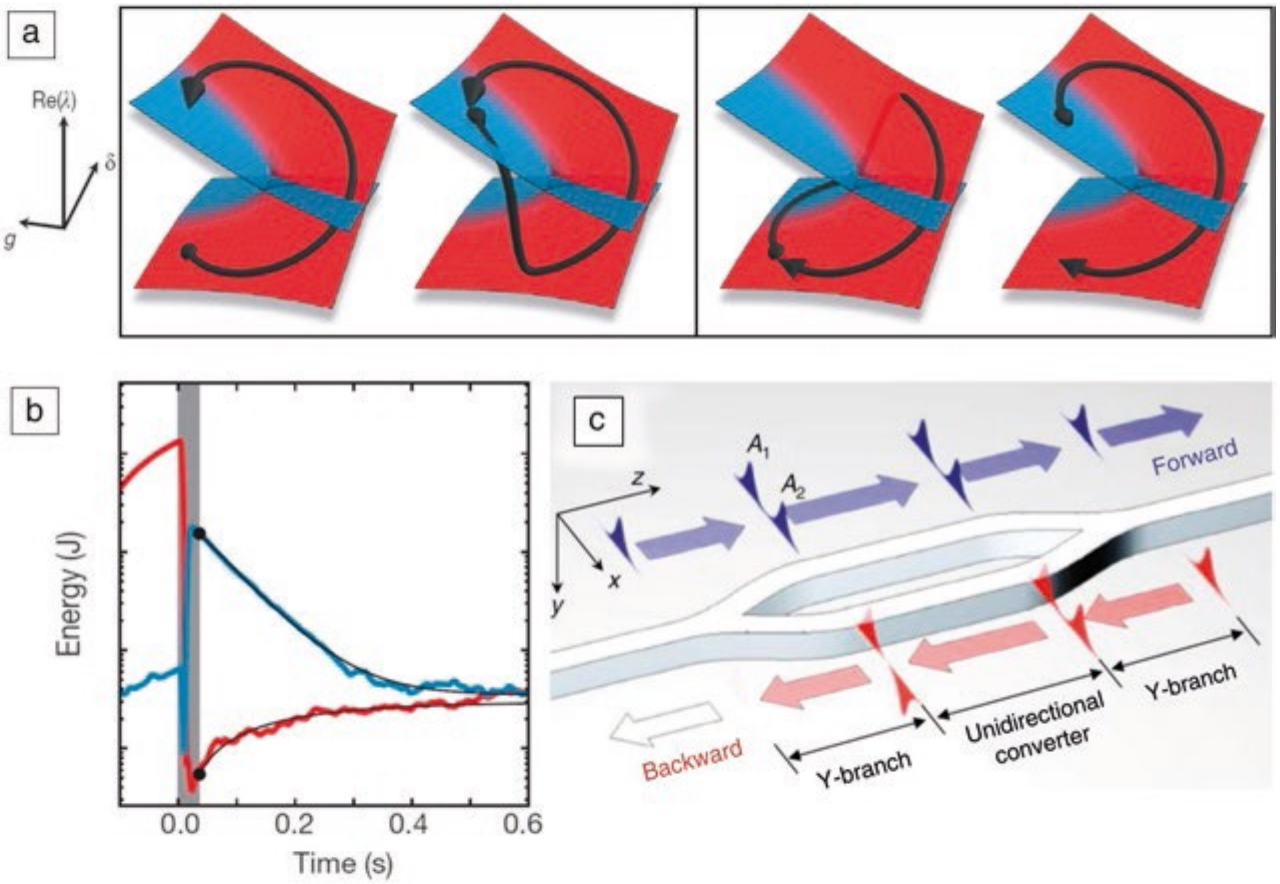
T. Kottos,
Nature Phys.
6, 166 (2010)

HERMITIAN vs NON-HERMITIAN PHOTONICS

- Real eigenvalues vs Complex eigenvalues.
- Eigenvalue degeneracies vs Exceptional Points (EP).
- Orthogonal eigenfunctions (spectral theorem)
  vs Coalescence of eigenfunctions.

- Mode distribution symmetric vs asymmetric/symmetric.
- Real eigenvalue surfaces vs intersecting Riemann sheets.
- Continuous eigenstate evolution vs “state flip”.
- Adiabaticity vs adiabatic theorem does not hold.

EXCEPTIONAL POINT: DYNAMICAL ENCIRCLING



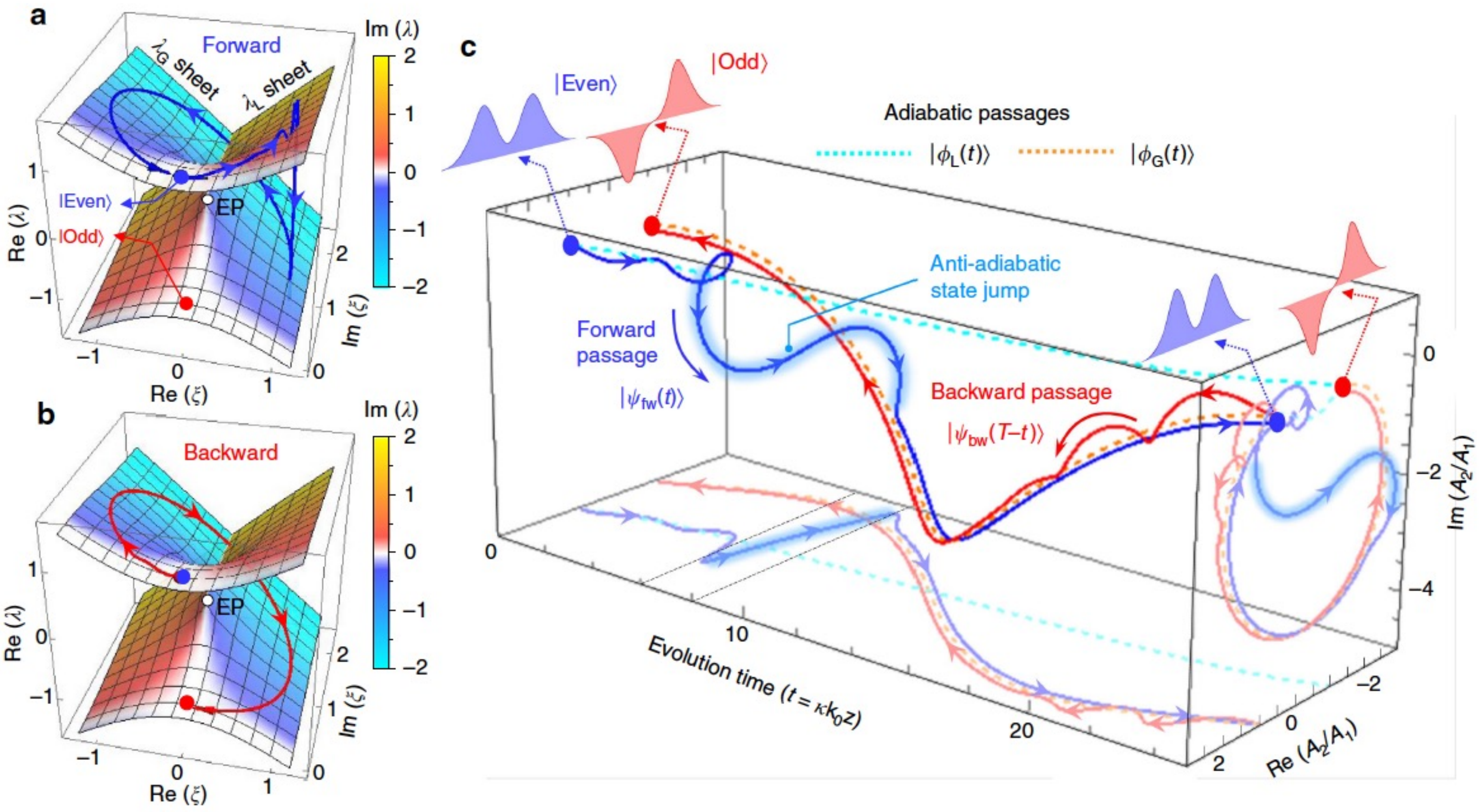
J. Doppler et al., Nature 537, 76 (2016)

H. Xu et al., Nature 537, 80 (2016)

Y. Choi et al., Nat. Commun. 8, 14154 (2017)

Time-asymmetric evolution:

Unidirectional Converter Region



PT- AND ANTI-PT SYMMETRY

- **PT Symmetric qubit**

$$(\mathcal{PT})^2 = +1$$

$$[H, \mathcal{PT}] = 0$$

$-\Omega(t)$ Phase function

$\rho_S(t)$ Reduced density matrix

B. Gardas, S. Deffner, A. Saxena,
Phys. Rev. A 94, 040101(R) (2016)

- **Anti-PT Symmetric qubit**

$$(\mathcal{PT})^2 = \pm 1$$

$$\{H, \mathcal{PT}\} = 0$$

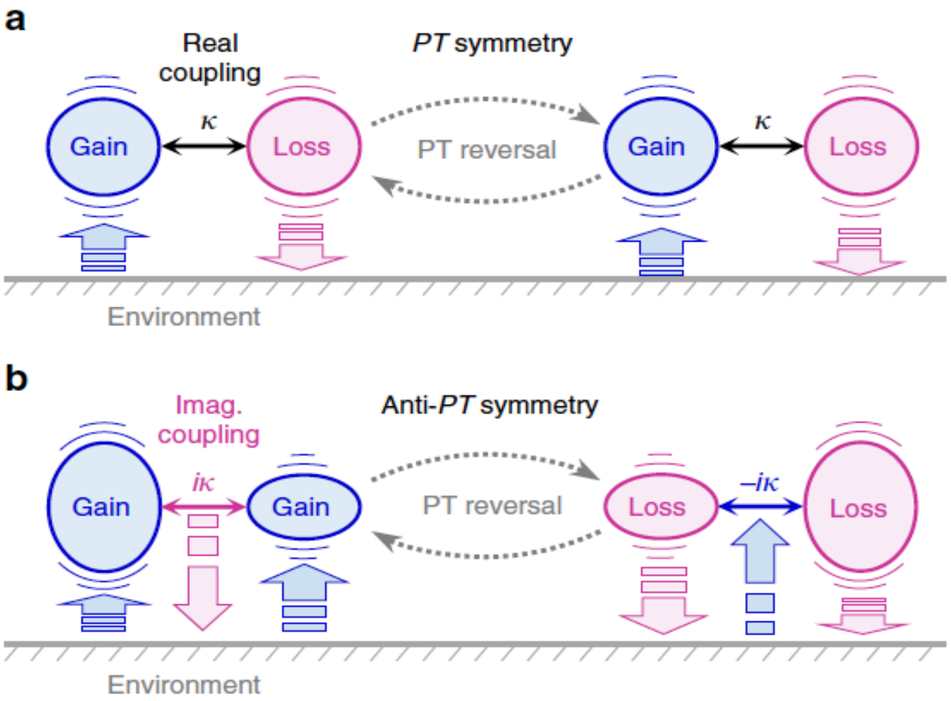
$$\Omega_2(t) - \Omega_1(t)$$

Time-dependent Dyson map

J. Cen and A. Saxena,
Phys. Rev. A 105, 022404 (2022)

$\mathcal{PT}$ and Anti-$\mathcal{PT}$ Coupled Oscillators

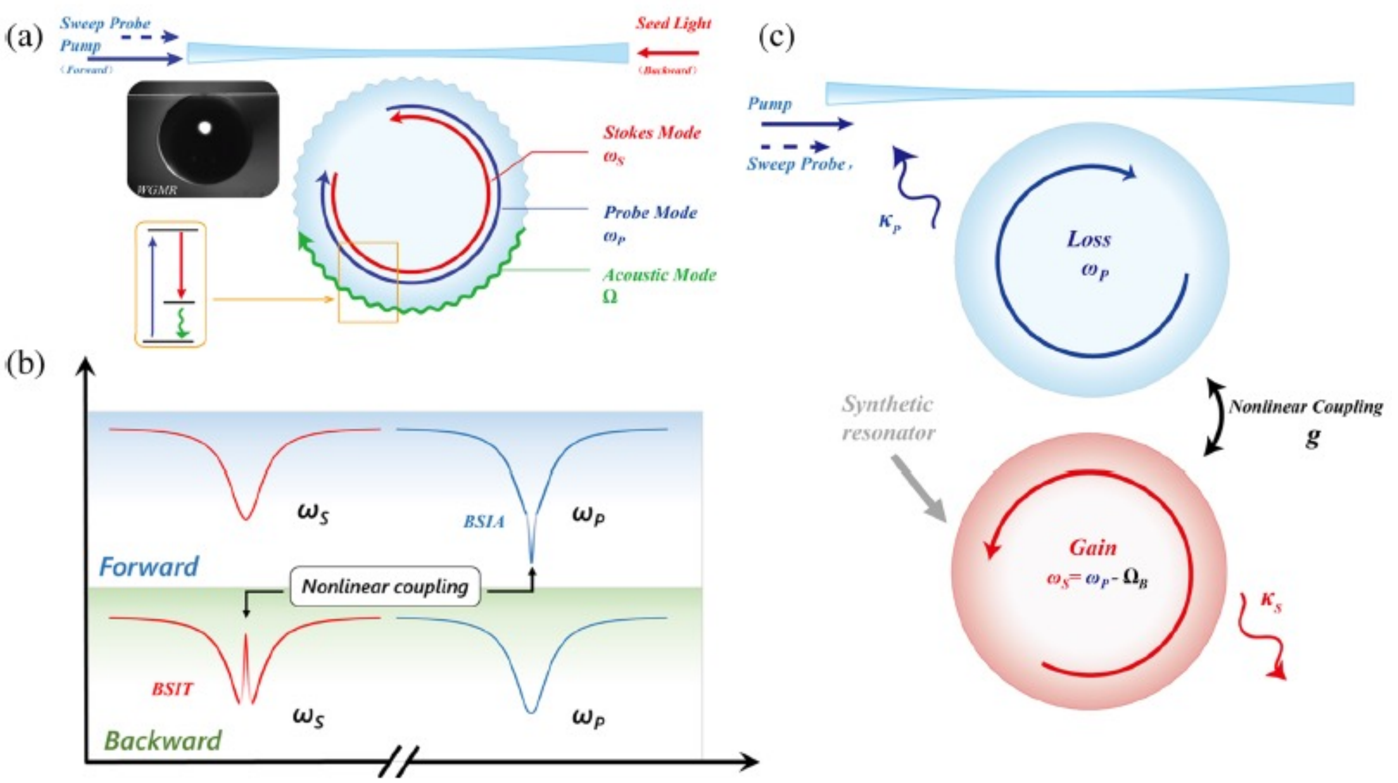
Choi et al. Nature Comm. 2018



$$[H, \mathcal{PT}] = 0$$

$$\{H, \mathcal{PT}\} = 0$$

Synthetic Anti-PT Optical Microcavity



F. Zhang et al., Phys. Rev. Lett. 124, 053901 (2020)

Anti-$\mathcal{PT}$-symmetry:

- L. Ge and H.E. Tureci, Phys. Rev. A **88**, 053810 (2013); photonics
- P. Peng et al., Nature Phys. **12**, 1139 (2016); flying atoms
- X. Wang et al., Opt. Exp. **24**, 4289 (2016); atomic lattices
- F. Yang et al., Phys. Rev. A **96**, 053845 (2017); dissipative optics
- Y. Choi et al., Nature Commun. **9**, 2182 (2018); circuit resonators
- Y.-L. Chuang et al., Opt. Exp. **26**, 21969 (2018); atomic systems
- V. V. Konotop and D. A. Zezyulin, Phys. Rev. Lett. **120**, 123902 (2018); optical coupler
- Q. Li et al., Optica **6**, 67 (2019); Lorentz dynamics
- X.-L. Zhang et al., Light: Sci. Appl. **8**, 88 (2019); asymmetric mode switching
- Y. Jiang et al., Phys. Rev. Lett. **123**, 193604 (2019); cold atoms
- S. Ke at al., Opt. Exp. **27**, 13858 (2019); optical waveguides
- … … … …

$\mathcal{PT}$-symmetry and Quantum Information

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **94**, 040101(R) (2016)

$\mathcal{PT}$-symmetric slowing down of decoherence

Bartłomiej Gardas,[1,2] Sebastian Deffner,[1,3,4] and Avadh Saxena[1,4]
[1] *Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*
[2] *Institute of Physics, University of Silesia, 40-007 Katowice, Poland*
[3] *Department of Physics, University of Maryland Baltimore County, Baltimore, Maryland 21250, USA*
[4] *Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*
(Received 19 July 2016; published 27 October 2016)

PHYSICAL REVIEW A **100**, 010102(R) (2019)

Rapid Communications

Eternal life of entropy in non-Hermitian quantum systems

Andreas Fring* and Thomas Frith†
Department of Mathematics, City, University of London, Northampton Square, London EC1V 0HB, United Kingdom

(Received 28 May 2019; published 26 July 2019)

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ELSEVIER

Controlling decoherence via $\mathcal{PT}$-symmetric non-Hermitian open quantum systems

Sanjib Dey*, Aswathy Raj, Sandeep K. Goyal
Department of Physical Sciences, Indian Institute of Science Education and Research Mohali, Sector 81, SAS Nagar, Manauli 140306, India

The System:

J. Cen & A. Saxena, Phys. Rev. A **105**, 022404 (2022)

$$H = H_S \otimes \mathbb{I}_B + \mathbb{I}_S \otimes H_B + H_S \otimes V_B$$

Hermitian

$$H_S^h = \begin{pmatrix} \alpha + \theta & \xi + i\delta \\ \xi - i\delta & -\alpha + \theta \end{pmatrix}$$

$\mathcal{PT}$-symmetric

$$H_S^{\mathcal{PT}} = \begin{pmatrix} \alpha + i\theta & \xi + i\delta \\ \xi - i\delta & \alpha - i\theta \end{pmatrix}$$

Anti-$\mathcal{PT}$-symmetric

$$H_S = \begin{pmatrix} \alpha + i\theta & \xi + i\delta \\ -\xi + i\delta & -\alpha + i\theta \end{pmatrix}$$

Bosonic bath

$$H_B = \sum_k \omega_k a_k^\dagger a_k$$

$$V_B = \sum_k \left(g_k a_k^\dagger + g_k^* a_k \right)$$

Similarity Transformation:

Hermitian

$$H_S^h = \begin{pmatrix} \alpha+\theta & \xi+i\delta \\ \xi-i\delta & -\alpha+\theta \end{pmatrix}$$

$$E_\pm = \theta \pm \omega_0$$

$$\omega_0 = \sqrt{\alpha^2+\xi^2+\delta^2}$$

$$\begin{pmatrix} -\omega_0+\theta & 0 \\ 0 & \omega_0+\theta \end{pmatrix}$$

$\mathcal{PT}$-symmetric

$$H_S^{\mathcal{PT}} = \begin{pmatrix} \theta+i\alpha & \xi+i\delta \\ \xi-i\delta & \theta-i\alpha \end{pmatrix}$$

$$T = \begin{pmatrix} \omega_0-\alpha & -\xi-i\delta \\ \omega_0+\alpha & \xi+i\delta \end{pmatrix}$$

$$\omega_0 = \sqrt{-\alpha^2+\xi^2+\delta^2}$$

$$\begin{pmatrix} -\omega_0+\theta & 0 \\ 0 & \omega_0+\theta \end{pmatrix}$$

Anti-$\mathcal{PT}$-symmetric

$$H_S = \begin{pmatrix} \alpha+i\theta & \xi+i\delta \\ -\xi+i\delta & -\alpha+i\theta \end{pmatrix}$$

$$E_\pm = i\theta \pm \omega_0$$

$$\omega_0 = \sqrt{\alpha^2-\xi^2-\delta^2}$$

$$\begin{pmatrix} -\omega_0+i\theta & 0 \\ 0 & \omega_0+i\theta \end{pmatrix}$$

Hermitian Qubit Dynamics: Similar for PT-symmetric qubit

$$\rho_S^{hD}(t) = \begin{pmatrix} \rho_{11}^{hD} & \rho_{12}^{hD} e^{2i\omega_0 t - i\omega_0\Omega(t)} e^{-\omega_0^2\gamma(t)} \\ \rho_{21}^{hD} e^{-2i\omega_0 t + i\omega_0\Omega(t)} e^{-\omega_0^2\gamma(t)} & \rho_{22}^{hD} \end{pmatrix}$$

$$\Omega(t) = 4\theta \int_0^\infty dw J(w) \frac{wt - \sin(wt)}{w^2},$$

$$\gamma(t) = 4 \int_0^\infty dw J(w) \frac{1 - \cos(wt)}{w^2} \coth\left(\frac{\beta w}{2}\right)$$

$$J(\omega) = J_0 \omega^{1+\mu} e^{\frac{-\omega}{\omega_c}}$$

$$\rho_S^h(t) = T^{-1} \rho_S^{hD}(t) [T^{-1}]^\dagger$$

Anti-PT-symmetric Qubit Dynamics:

Non-Hermitian Liouville-von Neumann equation

$$i\rho_t^D = H^D \rho^D - \rho^D \left(H^D\right)^\dagger$$

$$\rho^D = \eta^{-1}\rho^{hD}\eta^{-1} \qquad |\phi_i\rangle = \eta|\psi_i\rangle$$

$$\rho_S^D(t) = \rho_S^{hD}(t)e^{\Omega_\eta(t)}$$

$$\Omega_\eta(t) = 2\theta t + 2\theta^2 t^2 \int_0^\infty d\omega\, J(\omega) \coth\left(\frac{\beta\omega}{2}\right)$$

Anti-PT-symmetric Qubit Dynamics:

Time-dependent Dyson relation

$$H^D = \eta^{-1} h^D \eta - i\eta^{-1}\eta_t$$

$$H_S^D = -\omega_0\sigma_z + i\theta\mathbb{I}_S$$

$$\eta = e^{-\theta t(1+V_B)}$$

$$h^D = -\omega_0\sigma_z(1+V_B) + H_B + \theta t\widehat{V_B} - \theta^2 t^2 \Omega_k$$

$$\widehat{V_B} = \sum_k \omega_k\left(g_k a_k^\dagger - g_k^* a_k\right), \quad \Omega_k = \sum_k \omega_k |g_k|^2$$

Anti-PT-symmetric Qubit Dynamics:

$$\rho_S^{hD}(t) = \begin{pmatrix} \rho_{11}^{hD} & \rho_{12}^{hD} e^{2i\omega_0 t - i\omega_0[\Omega_2(t)-\Omega_1(t)]} e^{-\omega_0^2 \gamma(t)} \\ \rho_{21}^{hD} e^{-2i\omega_0 t + i\omega_0[\Omega_2(t)-\Omega_1(t)]} e^{-\omega_0^2 \gamma(t)} & \rho_{22}^{hD} \end{pmatrix}$$

$$\Omega_1(t) = 4\theta \int_0^\infty dw J(w) \frac{(1-\cos(wt))}{w^2}$$

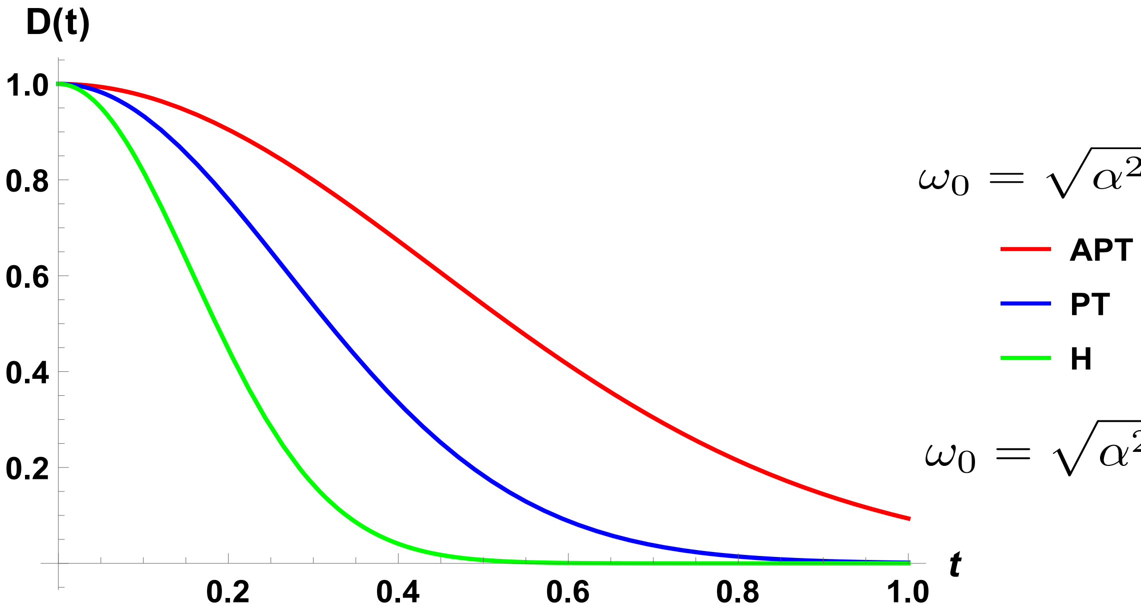
$$\Omega_2(t) = 2\theta t^2 \int_0^\infty dw J(w)$$

$$\rho^{hD} = \sum_i P_i |\phi\rangle\langle\phi| \qquad \rho^D = \sum_i P_i |\psi\rangle\langle\psi|$$

Decoherence:

$$D\,(t) = e^{-\omega_0^2 \gamma(t)}$$

$$\gamma(t) = 4\int_0^{\infty} d\omega\, J(\omega)\, \frac{1-\cos(\omega t)}{\omega^2} \coth(\frac{\beta\omega}{2})$$



$$\omega_0 = \sqrt{\alpha^2 - \xi^2 - \delta^2}$$

$$\omega_0 = \sqrt{\delta^2 + \xi^2 - \theta^2}$$

$$\omega_0 = \sqrt{\alpha^2 + \delta^2 + \xi^2}$$

von Neumann Entropy

Density matrix: $$\rho_S^{Dh}(t) = \frac{1}{2}\left[1 + \vec{v}(t)\,\vec{\sigma}\right].$$

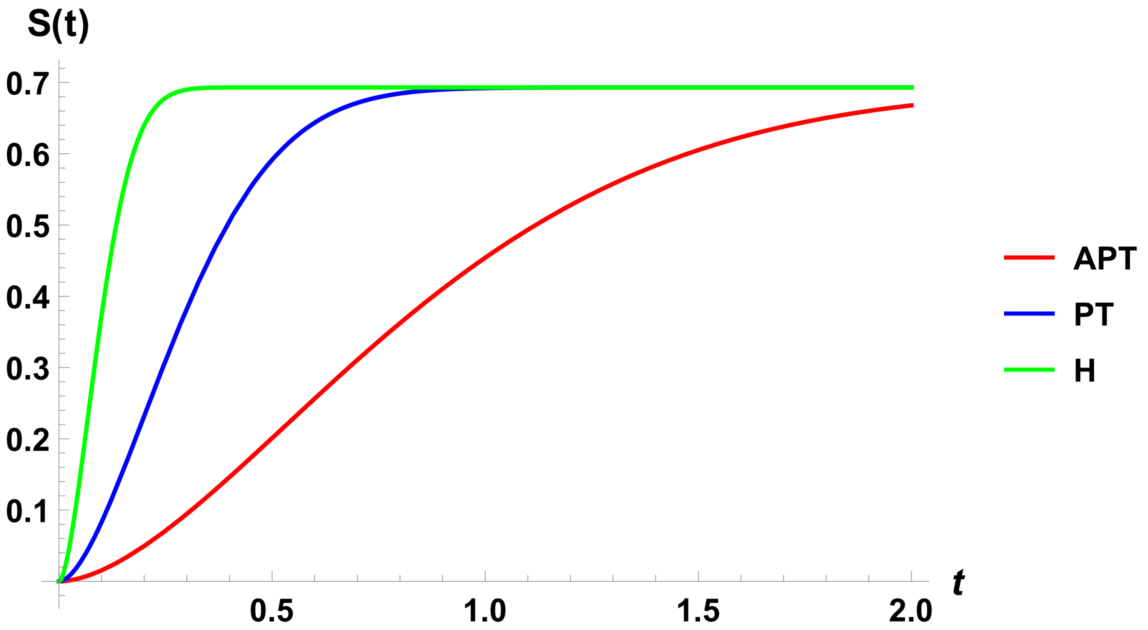
where $$v(t) = \sqrt{\langle\sigma_z\rangle^2 + (1 - \langle\sigma_z\rangle^2)e^{-2\omega_0^2\gamma(t)}}.$$

Entropy: $$\boxed{S(t) = -Tr_S\left[\rho_S^{Dh}(t)\ln\rho_S^{Dh}(t)\right].}$$

$$S(t) = \ln 2 - \frac{1}{2}\left(1 + e^{-\omega_0^2\gamma(t)}\right)\ln\left(1 + e^{-\omega_0^2\gamma(t)}\right) - \frac{1}{2}\left(1 - e^{-\omega_0^2\gamma(t)}\right)\ln\left(1 - e^{-\omega_0^2\gamma(t)}\right).$$

von Neumann Entanglement Entropy:

$$S(t) = \ln 2 - \frac{1}{2}[1 + D(t)]\ln[1 + D(t)] - \frac{1}{2}[1 - D(t)]\ln[1 - D(t)]$$



FISHER INFORMATION

Kullback-Leibler divergence:

$$D_{KL}\left(K,t\right)=Tr\left[\rho_S^{Dh}\left(\widetilde{K},t\right)\ln\rho_S^{Dh}\left(\widetilde{K},t\right)\right]-Tr\left[\rho_S^{Dh}\left(\widetilde{K},t\right)\ln\rho_S^{Dh}\left(K,t\right)\right]$$

Fisher entropy (with T):

$$S_f\left(\beta,t\right)=\frac{\partial^2}{\partial\widetilde{\beta}^2}D_{KL}\left(\widetilde{\beta},t\right)\Big|_{\widetilde{\beta}=\beta},$$

$$=\frac{\omega_0^4}{2}\left[\coth\left(\omega_0^2\gamma\left(\beta,t\right)\right)-1\right]\left(\frac{\partial}{\partial\beta}\gamma\left[\beta,t\right]\right)^2,$$

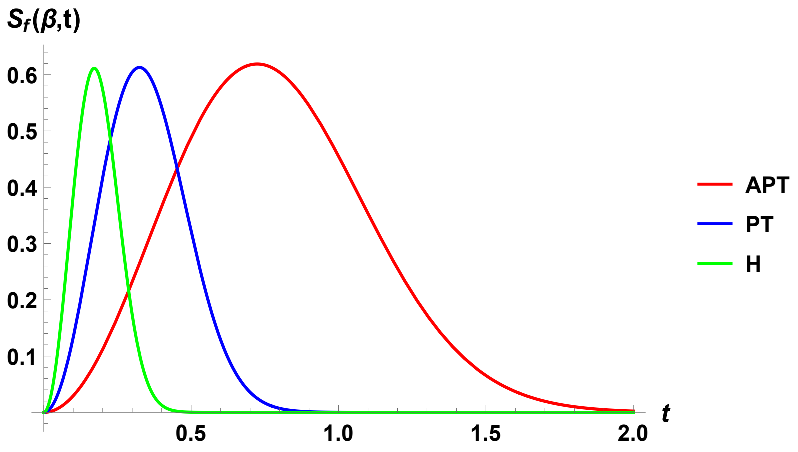
Fisher entropy (withω_0):

$$S_f\left(\omega_0,t\right)=\frac{\partial^2}{\partial\widetilde{\omega_0}^2}D_{KL}\left(\widetilde{\omega_0},t\right)\Big|_{\widetilde{\omega_0}=\omega_0},$$

$$=2\omega_0^2\left[\coth\left(\omega_0^2\gamma\left(\omega_0,t\right)\right)-1\right]\gamma^2\left(\omega_0,t\right).$$

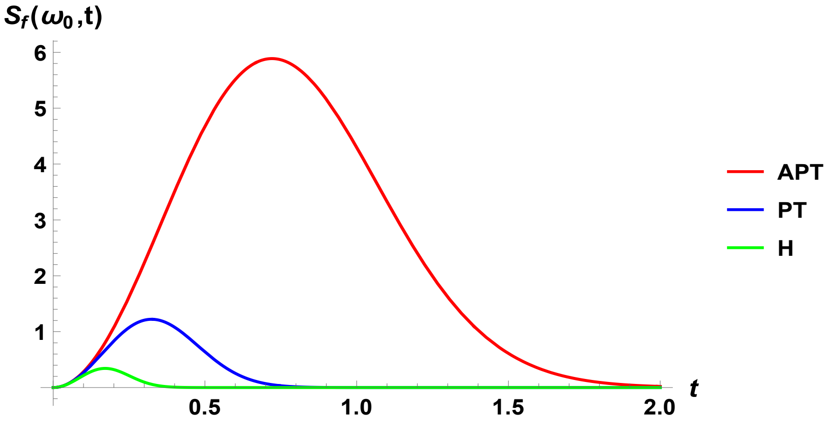
Fisher Information:

$$S_f(\beta,t) = \frac{\omega_0^4}{2}\{\coth[\omega_0^2\gamma(\beta,t)] - 1\}\left[\frac{\partial}{\partial\beta}\gamma(\beta,t)\right]^2$$



with Inverse T

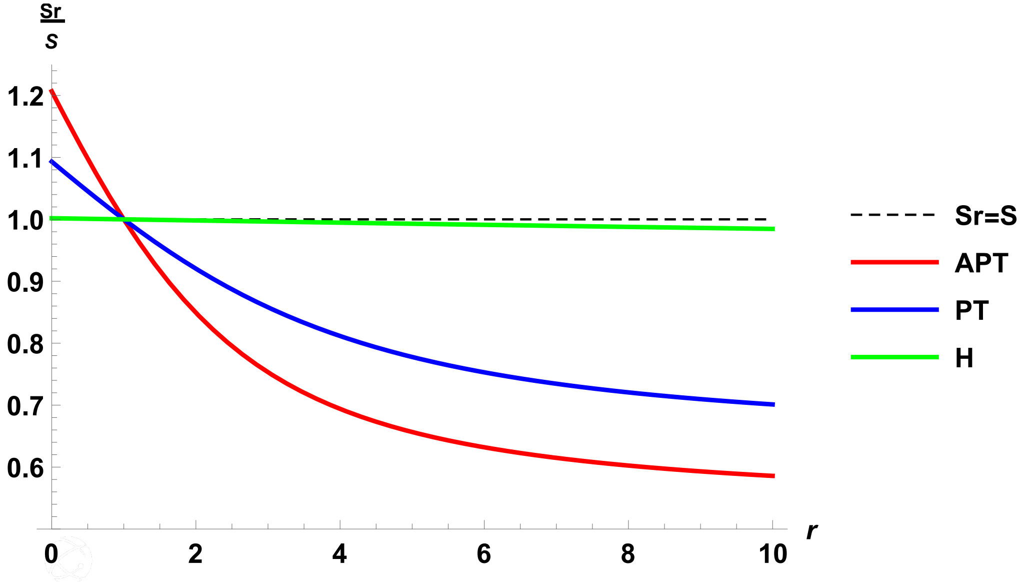
$$S_f(\omega_0,t) = 2\omega_0^2\{\coth[\omega_0^2\gamma(t)] - 1\}\gamma^2(t)$$

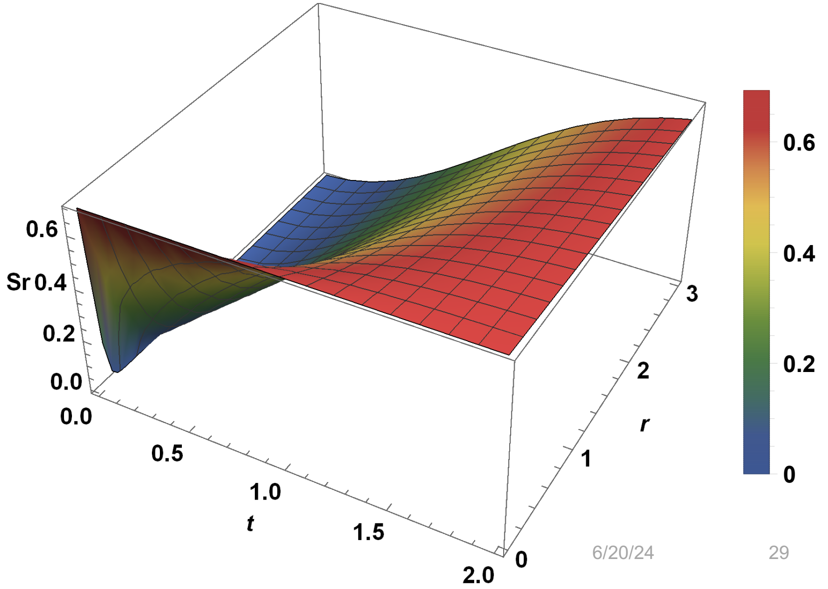


with ω_0

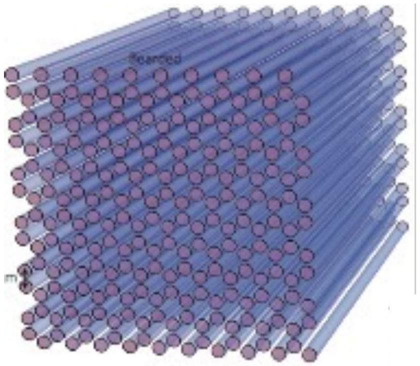
RENYI ENTANGLEMENT ENTROPY

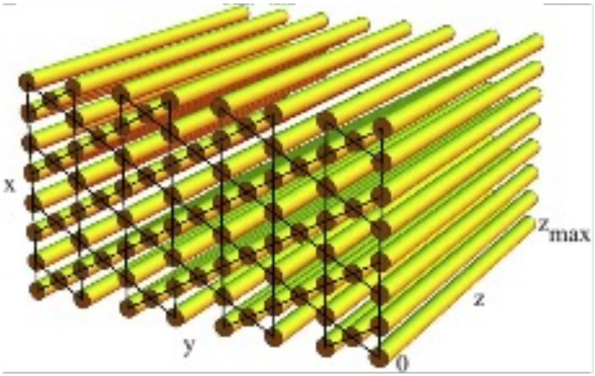
$$S_r(r,t) = \frac{\ln Tr\left\{\left[\rho_S^{Dh}(t)\right]^r\right\}}{1-r}$$
$$= \frac{r\ln 2}{1-r} + \frac{1}{1-r}\ln\left\{\left[1+e^{-\omega_0^2\gamma(t)}\right]^r + \left[1-e^{-\omega_0^2\gamma(t)}\right]^r\right\}$$

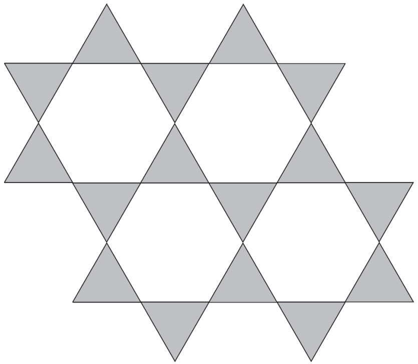




Discrete System: From photonic graphene to photonic Kagome

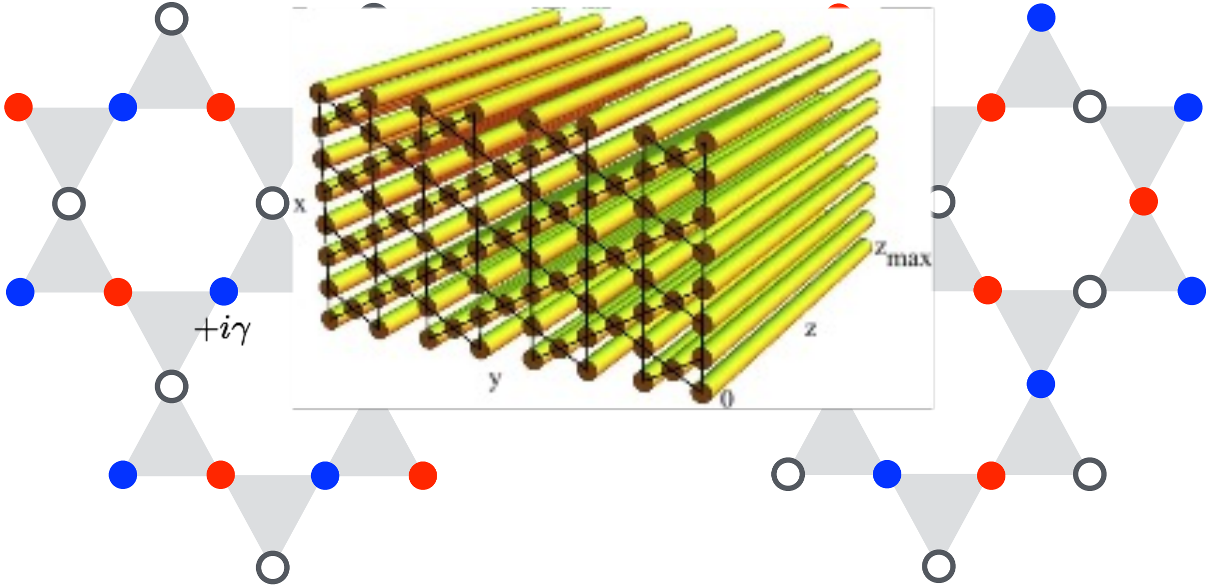




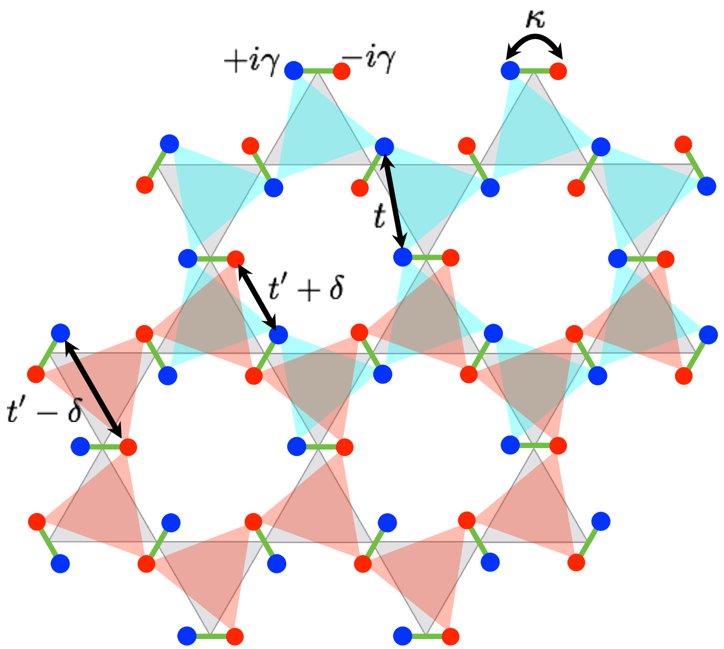


PT-symmetric Kagome lattices?

- $q = 0$ arrangement
- $\sqrt{3} \times \sqrt{3}$ arrangement



PT-symmetric Kagome dimer lattice



- Dimers at the kagome sites
- Two twisted kagome lattices
- Coupled mode equations:

$$i\frac{da_n}{dz} = +i\gamma\, a_n + \kappa\, b_n + \sum_m^{\mathrm{NN}} \left[t\, a_m + (t' \pm \delta)\, b_m\right],$$

$$i\frac{db_n}{dz} = -i\gamma\, b_n + \kappa\, a_n + \sum_m^{\mathrm{NN}} \left[t\, b_m + (t' \pm \delta)\, a_m\right].$$

G.W. Chern, A. Saxena, Opt. Lett. 40, 5806 (2015)

PT-symmetric Kagome: Band structure

- Fourier transform:

$$i\frac{d}{dz}\begin{pmatrix} a_{1,\mathbf{k}} \\ a_{2,\mathbf{k}} \\ a_{3,\mathbf{k}} \\ b_{1,\mathbf{k}} \\ b_{2,\mathbf{k}} \\ b_{3,\mathbf{k}} \end{pmatrix} = \begin{pmatrix} P(\mathbf{k}) & Q(\mathbf{k}) \\ Q(\mathbf{k})^* & P(\mathbf{k})^* \end{pmatrix} \cdot \begin{pmatrix} a_{1,\mathbf{k}} \\ a_{2,\mathbf{k}} \\ a_{3,\mathbf{k}} \\ b_{1,\mathbf{k}} \\ b_{2,\mathbf{k}} \\ b_{3,\mathbf{k}} \end{pmatrix}$$

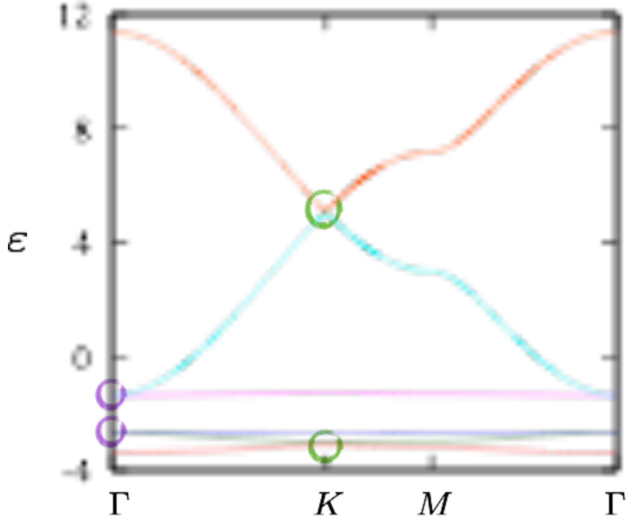
$$P(\mathbf{k}) = \begin{pmatrix} i\gamma & tc_3 & tc_2 \\ tc_3 & i\gamma & tc_1 \\ tc_2 & tc_1 & i\gamma \end{pmatrix},$$

$$Q(\mathbf{k}) = \begin{pmatrix} \kappa & t'c_3 - i\delta s_3 & t'c_2 - i\delta s_2 \\ t'c_3 + i\delta s_3 & \kappa & t'c_1 + i\delta s_1 \\ t'c_2 + i\delta s_2 & t'c_1 - i\delta s_1 & \kappa \end{pmatrix}.$$

$$c_{1,2} = 2\cos(\tfrac{k_x \pm \sqrt{3}k_y}{4}) \qquad c_3 = 2\cos(\tfrac{k_x}{2})$$

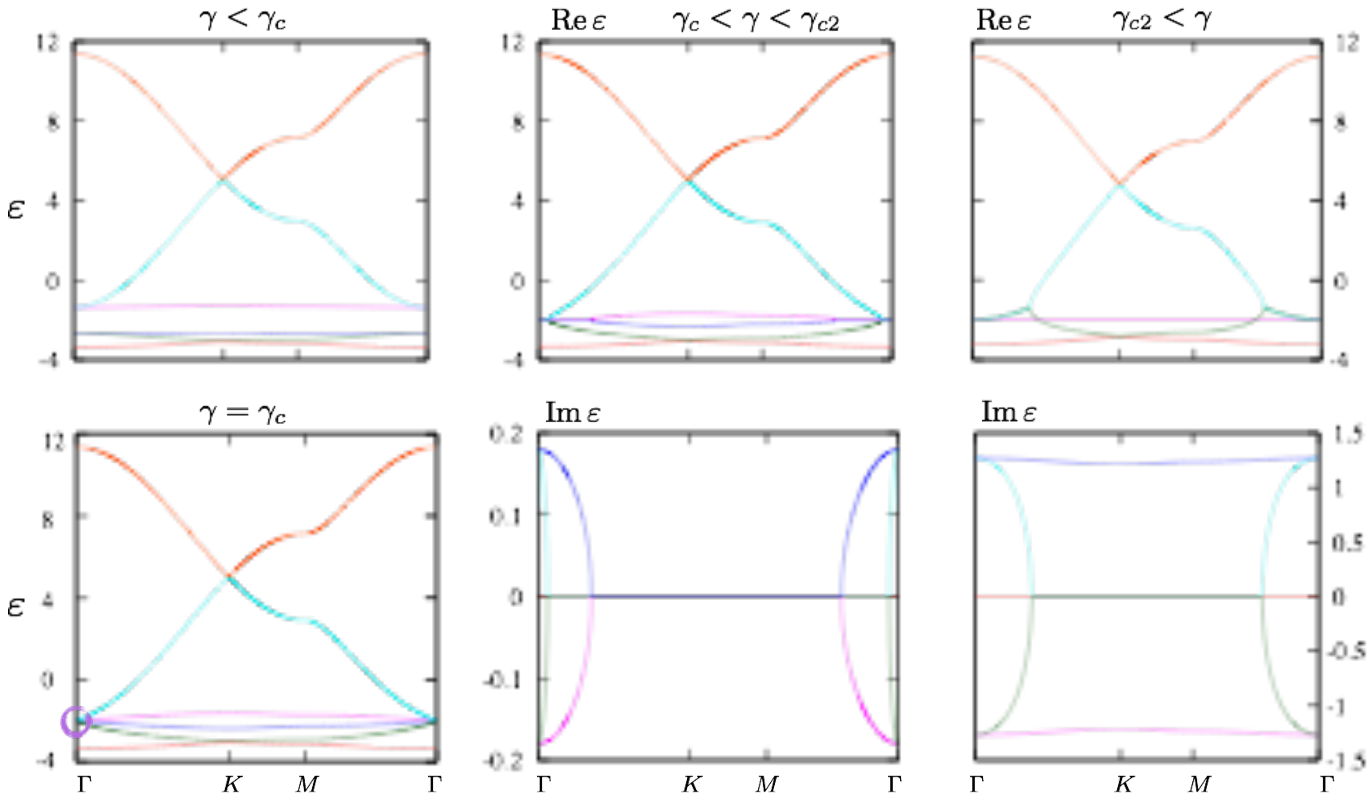
$$s_{1,2} = 2\sin(\tfrac{k_x \pm \sqrt{3}k_y}{4}) \qquad s_3 = 2\sin(\tfrac{k_x}{2})$$

$\gamma < \gamma_c$: PT-exact phase

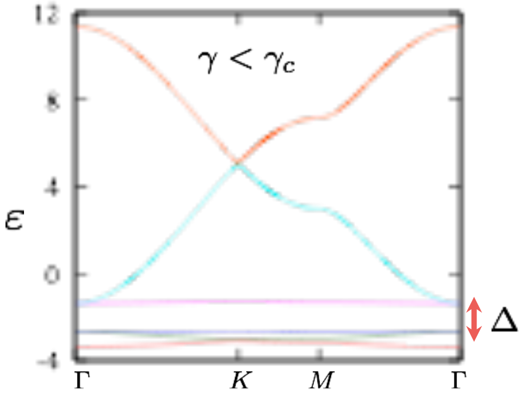


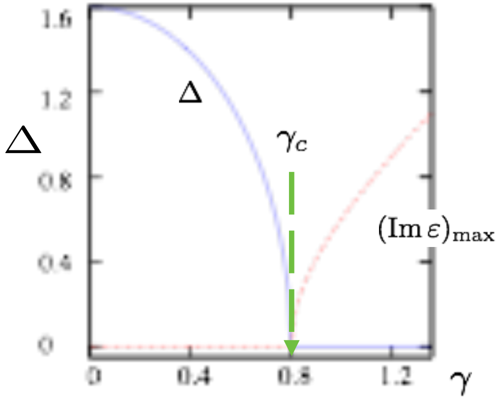
- Two sets of Dirac points
- Two topological quadratic points
- Two nearly flat bands

PT-symmetric Kagome: Band structure



PT-symmetric kagome: Petermann Coefficient



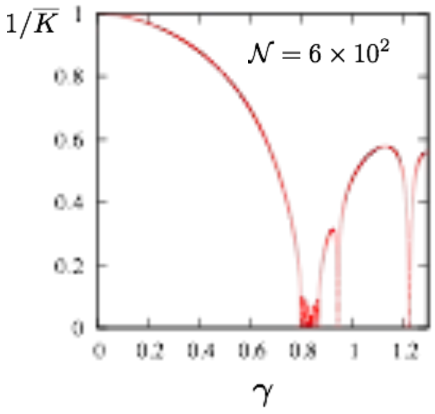


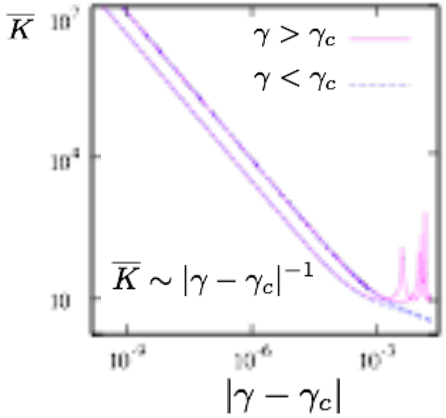
- Petermann factor:

$$K_{nm} = \langle L_n | L_m \rangle \langle R_m | R_n \rangle$$

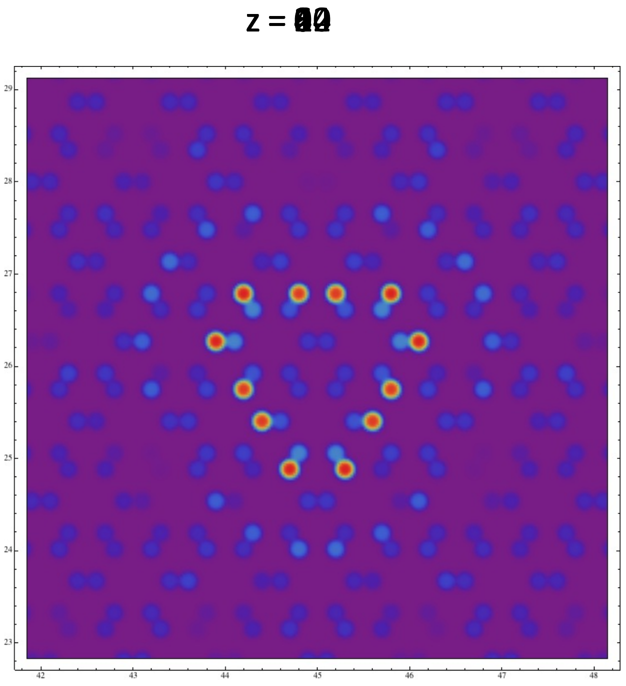
- Diagonal average

$$\overline{K} = \frac{1}{\mathcal{N}} \sum_{n=1}^{\mathcal{N}} K_{nn}$$

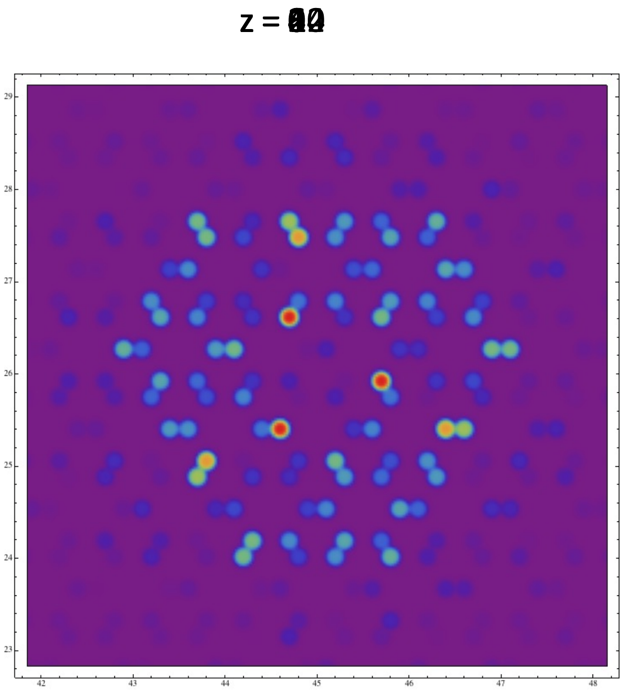




PT-symmetric kagome: Linear beam dynamics

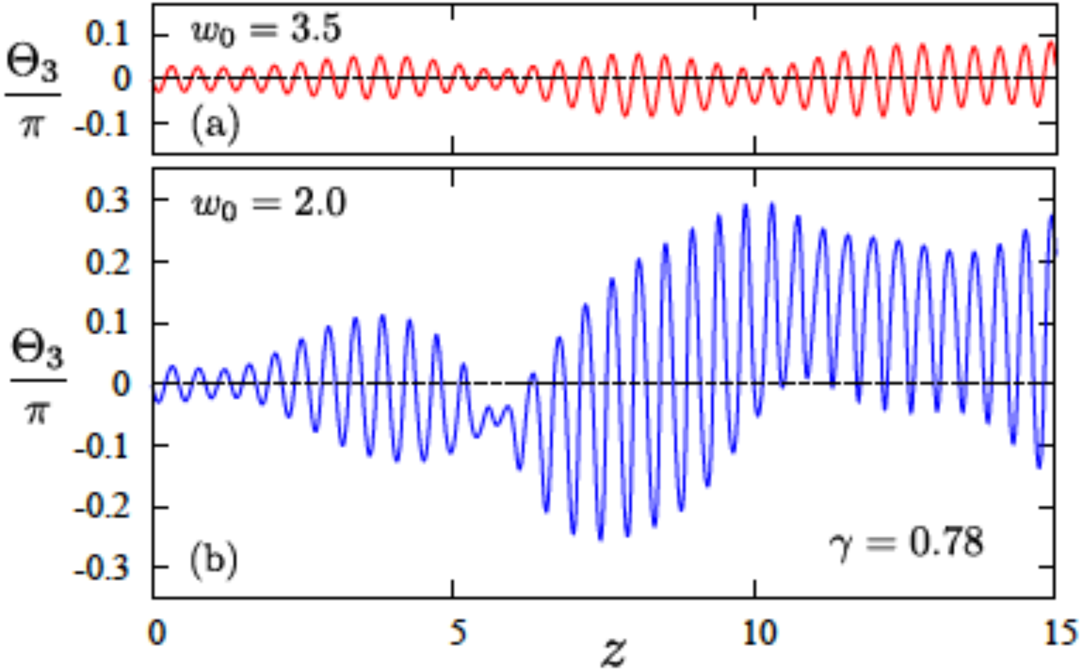


$\gamma = 0$



$0 < \gamma < \gamma_c$

Oscillatory Power Rotation



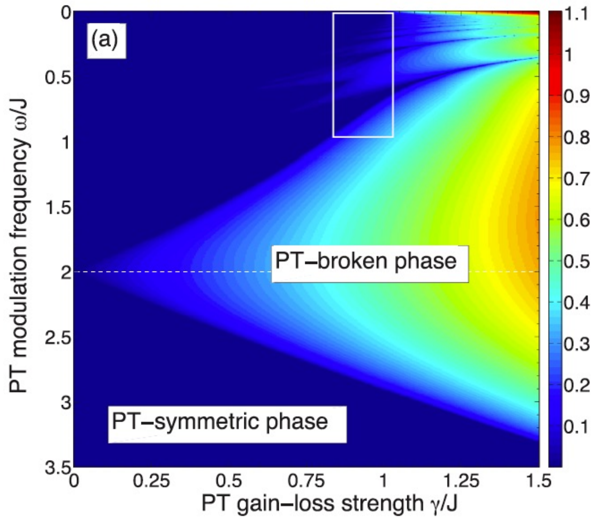
Wide beam

Narrow beam

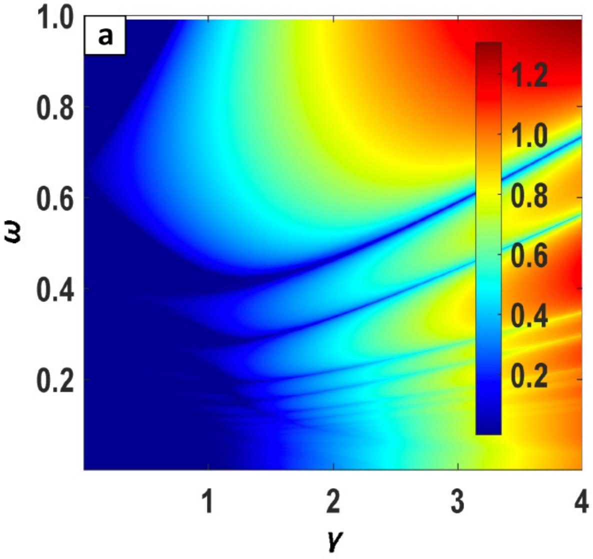
PT-Symmetric Kagome Lattice $\gamma = 0.78$

STABILITY OF NON-HERMITIAN HAMILTONIANS WITH DIFFERENT

PERIODICITIES USING FLOQUET THEORY



Single-Drive



Double-Drive

J. Cen, Y.N. Joglekar, A. Saxena, Phys. Rev. Res. 6, 013167 (2024)

Double-Drive Floquet Formalism:

Hamiltonian: $H(t) = \mathbf{A}(t) \cdot \sigma + i\mathbf{B}(t) \cdot \sigma$ $\quad \sigma = (X, Y, Z)$

$H(t) = H(t+T)$ $\quad \mathbf{A} \cdot \mathbf{B} = 0$

Floquet Hamiltonian: $H_F = H(t) - i\hbar\partial_t$ $\quad \lambda_k(t)$

$H_F(\omega_1, \omega_2)$ $\quad \epsilon_{n\alpha} = \epsilon_\alpha + n\omega$ $\quad \omega = 2\pi/T$

Time-evolution Operator: $G(T) = \mathcal{T}\exp(-i\int_0^T H(t')dt')$ Time-ordered product

PT-SYMMETRIC MODELS:

$$H(t) = JX + \gamma \left\{ \cos(\omega t) Y - i \cos(\beta \omega t) Z \right\}$$

$$\begin{aligned}
H_\ell &= JX + \gamma (\mathbf{v}_y \cdot \mathbf{l}) Y - i\gamma (\mathbf{v}_z \cdot \mathbf{l}) Z, \\
\mathbf{v}_y &= \big(\underbrace{1 \cdots 1}_{\beta} \underbrace{-1 \cdots -1}_{2\beta} \underbrace{1 \cdots 1}_{\beta} \big), \\
\mathbf{v}_z &= \big(\underbrace{1, -1, -1, 1,}_{\beta \text{ copies}} \cdots, 1, -1, -1, 1 \big), \\
\mathbf{l} &= (0, \cdots \underbrace{1}_{\ell\text{th entry}} \cdots, 0)
\end{aligned}$$

$$G(T) = \prod_{\ell=1}^{4\beta} e^{-iH_\ell T/4\beta} = \cos(\epsilon_F T)\, \mathbb{I}_2 - i \sin(\epsilon_F T)\, (\mathbf{n}_F \cdot \sigma)$$

Conclusions (non-Hermitian qubits)

- Qubits with $\mathcal{PT}$/anti-$\mathcal{PT}$-symmetries show promising results:
  - Decoherence
  - Entanglement entropy
  - Fisher information

- Reduced density matrix for non-Hermitian $\mathcal{PT}$/anti-$\mathcal{PT}$-symmetric systems

- $\mathcal{PT}$/anti-$\mathcal{PT}$-symmetric time-dependent systems (to be explored)

Non-Hermitian PT Symmetry is ubiquitous!

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- … … …

NON-

QUANTUMHERMITIAN

THANKS!