Inertial Quantum Sensors at the Interface of Relativity

From Quantum to Cosmology – 7th June 2024 – Los Alamos

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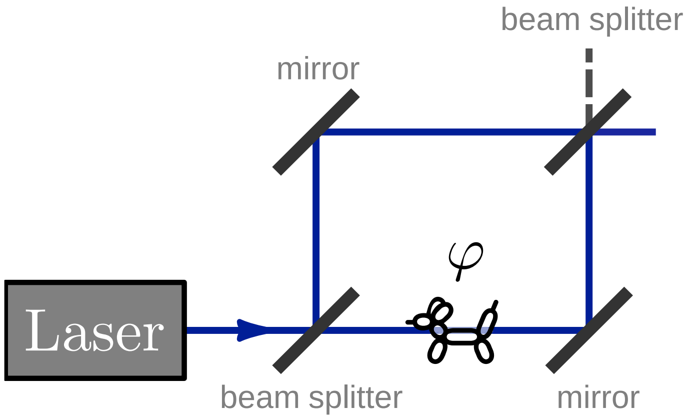
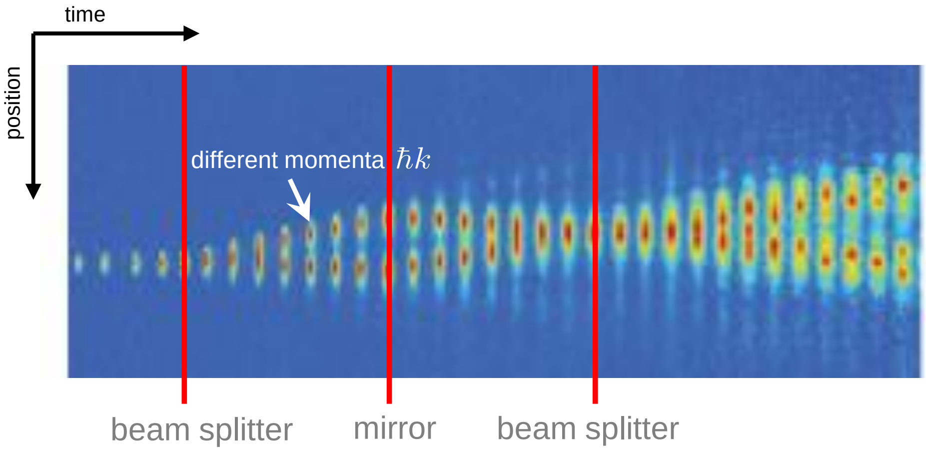
Introduction into Atom Interferometry



New J. Phys. **26**, 023018



Phys. Rev. Lett. **110**, 093602

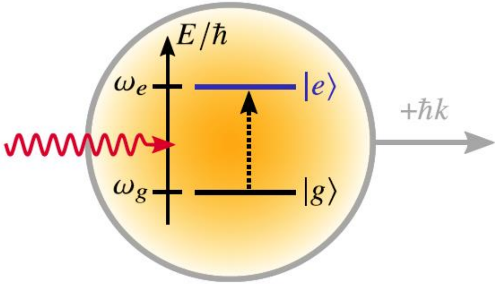
- optical Mach-Zehnder interferometer (MZI)
- (coherent) light is manipulated by matter
- object properties imprinted on interference signal

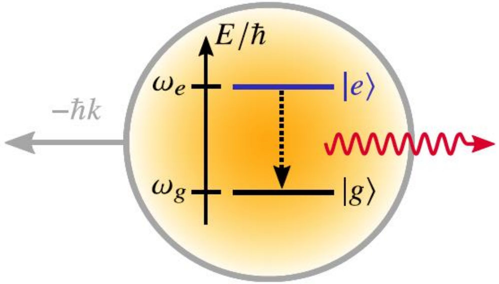
→ sensing of these properties

- atomic MZI
- (coherent) matter is manipulated by light
- matter has mass!

→ sensing of inertial forces possible

Manipulating the Centre-of-Mass Motion





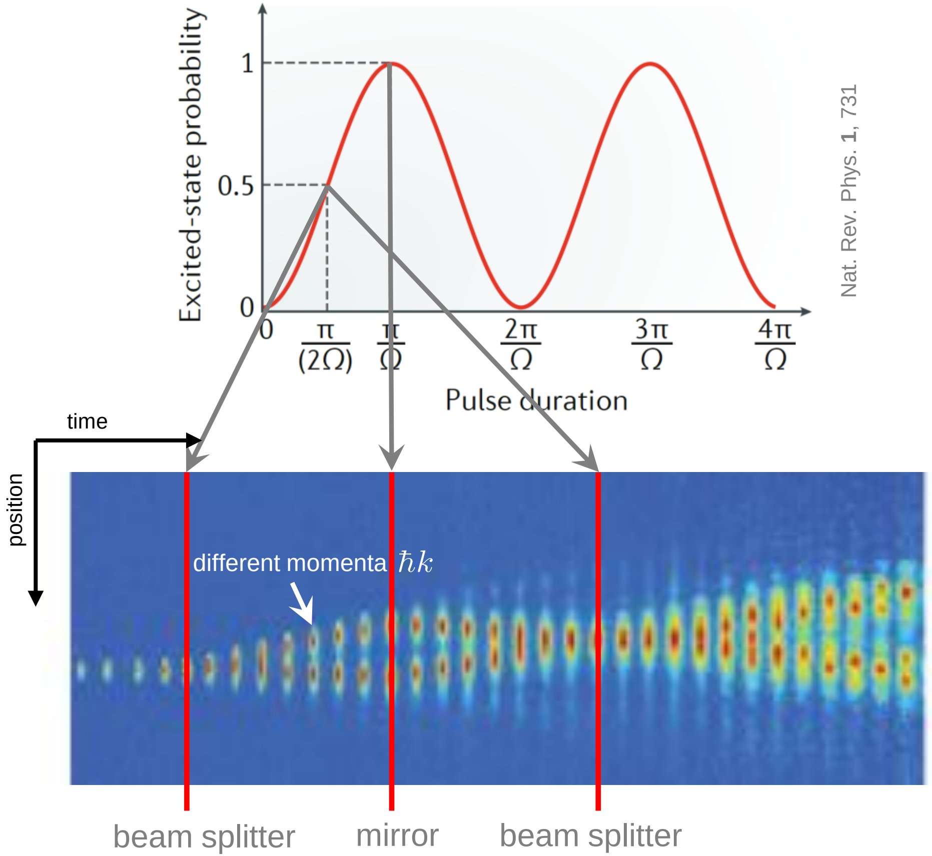
- stimulated absorption and emission
- incoming light with energy and momentum
- energy and momentum conservation

→ manipulation of internal and external degrees of freedom

- laser phase ϕ_L is imprinted
- drive Rabi oscillation

Beam Splitter and Mirror

- induced Rabi oscillation
  between different states/momenta



Nat. Rev. Phys. **1**, 731

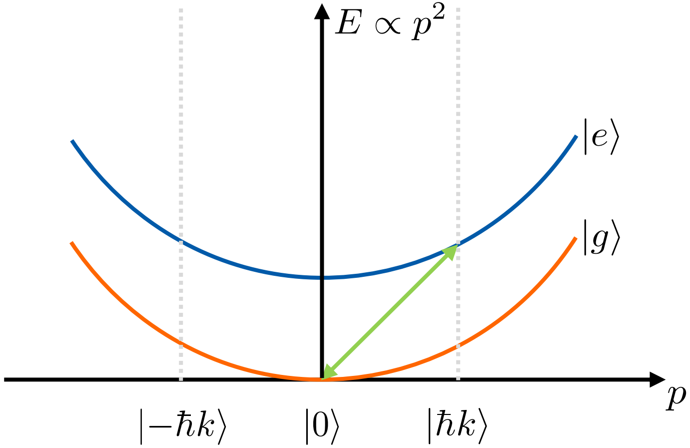
Phys. Rev. Lett. **110**, 093602

- vary interaction time (or Rabi frequency)

→ beam splitter or mirror pulse

Different Atomic Diffraction Schemes

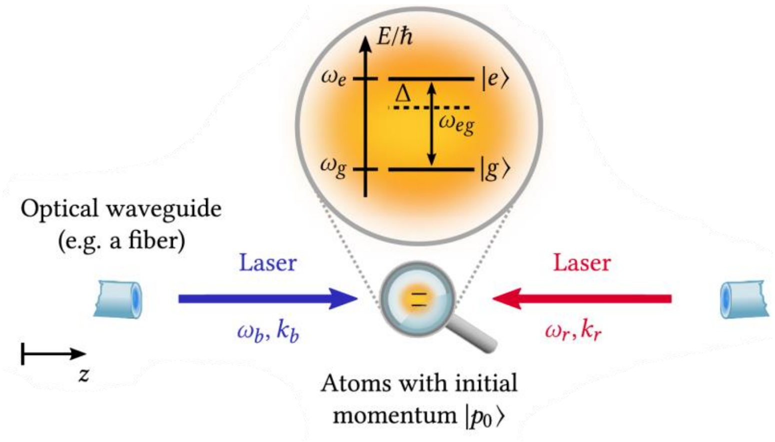
- distinguish between change of momentum with and without changing the internal state



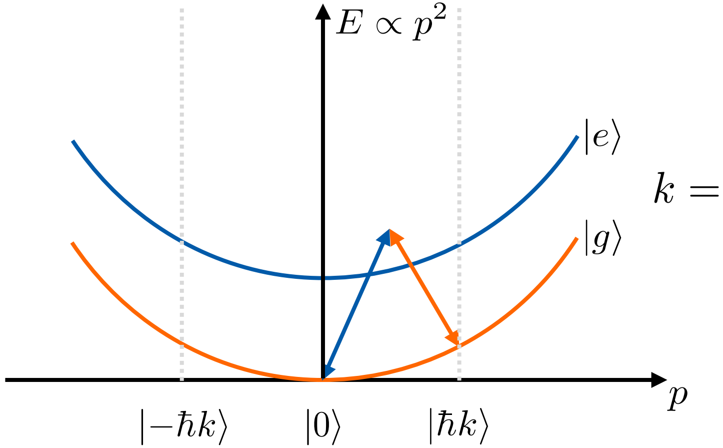
- state changing: single-photon transitions and Raman diffraction (two photons)

Different Atomic Diffraction Schemes

- distinguish between change of momentum with and without changing the internal state



credit Alexander Friedrich



$$k = k_b + k_r$$

- state changing: single-photon transitions and Raman diffraction (two photons)
- state preserving: single and double Bragg diffraction

- if resonance not hit exactly → various loss effects to unwanted paths (today we ignore this)

Measuring *g* with Atom Interferometry

- typical interference signal

phase

$$I_g = \frac{1}{2}\left(\mathcal{A} + \mathcal{V}\cos\varphi\right)$$

amplitude visibility

- momentum changes in gravity

→ compensate with chirping, i. e.

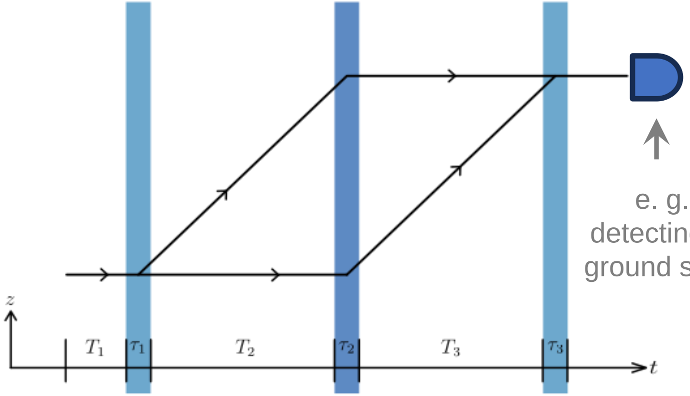
$$\phi_L(t) = \phi_{L,\text{off}} - kct + k\alpha t^2/2$$

chirping rate

- phase of closed MZI contains information about *g*

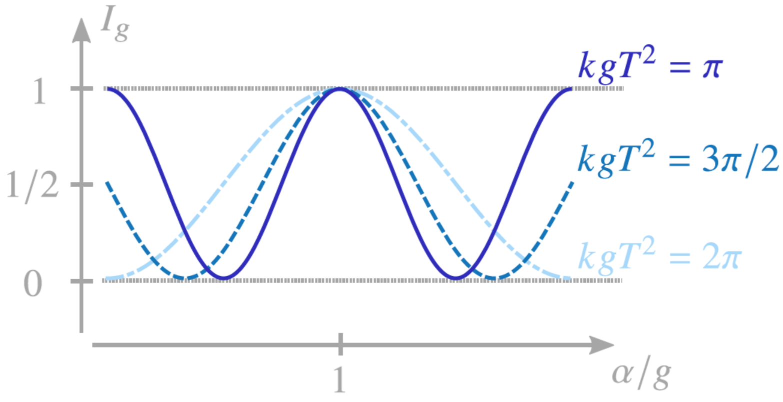
$$\varphi = \Delta\phi - kgT^2 = (\alpha/g - 1)kgT^2$$

$$\phi_{L,2} - 2\phi_{L,1} + \phi_{L,0}$$



e. g. detecting in ground state

Bachelor thesis, M. Dietrich



Book Chapter, Jump Start into Atom Interferometry

Measuring *g* with Atom Interferometry

- typical interference signal

$$I_g = \frac{1}{2}\left(\underset{\substack{\uparrow\\ \text{amplitude}}}{\mathcal{A}} + \underset{\substack{\nwarrow\\ \text{visibility}}}{\mathcal{V}} \cos \overset{\substack{\text{phase}\\ \swarrow}}{\varphi}\right)$$

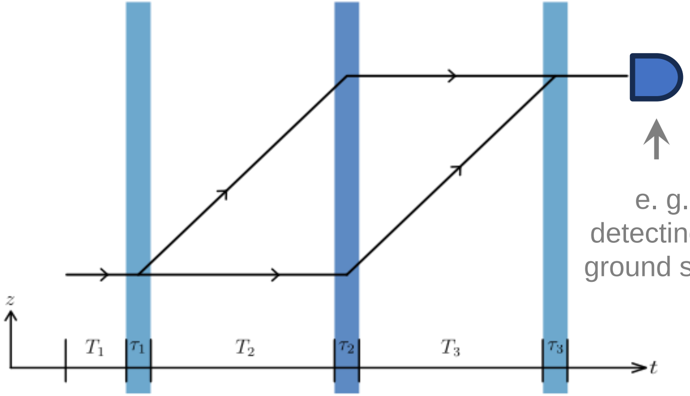
- momentum changes in gravity

→ compensate with chirping, i. e.

$$\phi_L(t) = \phi_{L,\mathrm{off}} - kct + k\underset{\substack{\uparrow\\ \text{chirping rate}}}{\alpha} t^2/2$$

- phase of closed MZI contains information about *g*

$$\varphi = \underset{\substack{\uparrow\\ \phi_{L,2} - 2\phi_{L,1} + \phi_{L,0}}}{\Delta\phi} - kgT^2 = (\alpha/g - 1)kgT^2$$



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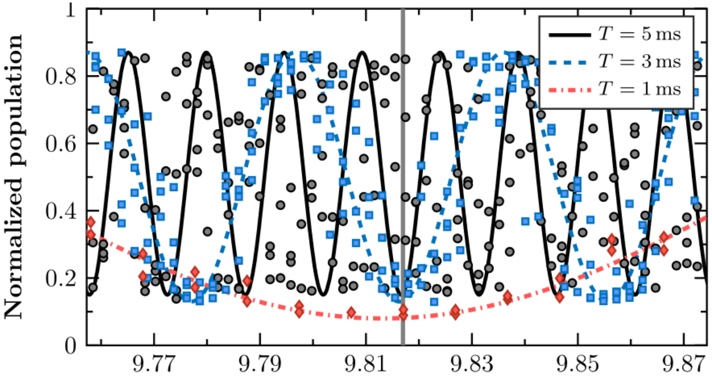


Phys. Rev. Lett. **117** 203003

Differential Measurements

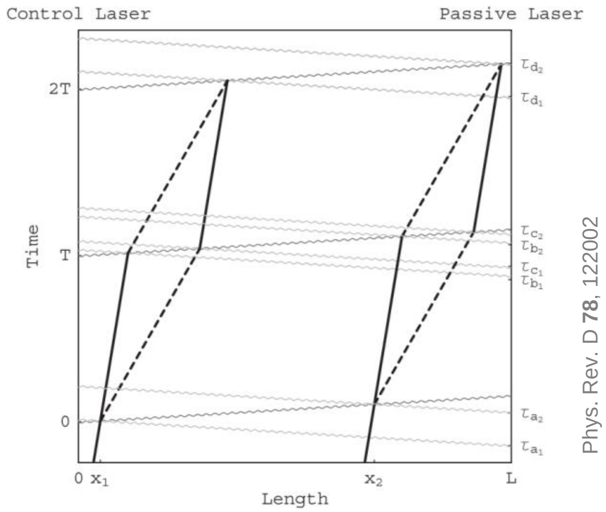
- differential measurements often useful to investigate small effects / perturbations

- gravitational-wave detection

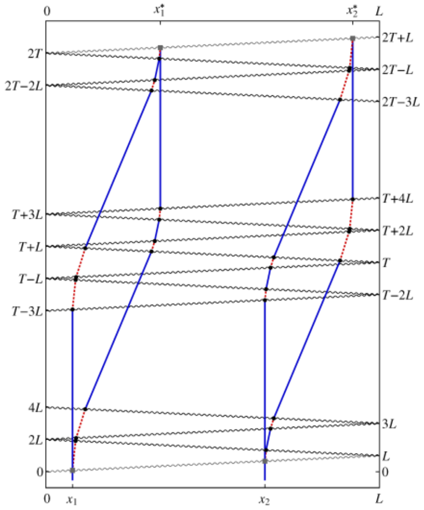
  Phys. Rev. D **78**, 122002

- dark-matter detection

  Phys. Rev. D **97**, 075020



Phys. Rev. D **78**, 122002



Phys. Rev. D **97**, 075020

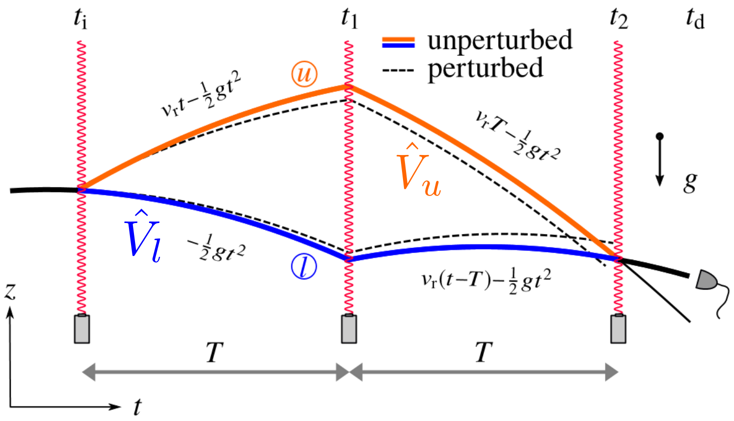
- single-photon transitions better suited for differential measurements (laser phase noise, locking of two different lasers)

  Phys. Rev. Lett. **110**, 171102

Atom Interferometers for Metrology

- ► interested in high-precision measurements → use phase of atom interferometers

→ perturbative operator approach (no path integrals needed!) cf. Phys. Rev. A **101**, 053615



Phys. Rev. A **101**, 053615

1. separation of branches

$$|\psi_0\rangle \mapsto \frac{1}{\sqrt{2}}\left(\hat{U}_l + \hat{U}_u\right)|\psi_0\rangle$$

propagation along branches

2. overlap

contrast

phase

$$\langle\psi_0|\hat{U}_l^\dagger\hat{U}_u|\psi_0\rangle = C\exp(\mathrm{i}\varphi)$$

to be evaluated at classical trajectories

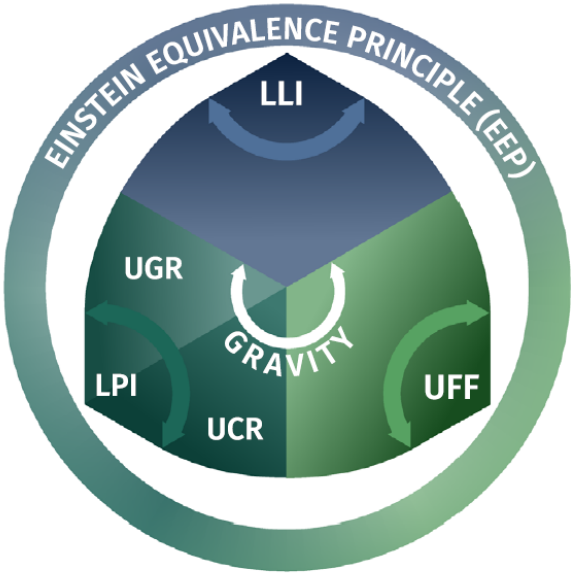
3. path-dependent perturbation

$$\hat{V}_i \quad \Rightarrow \quad \varphi^{(1)}_{\mathrm{pert}} = -\frac{1}{\hbar}\int \mathrm{d}t(\hat{V}_u - \hat{V}_l)$$

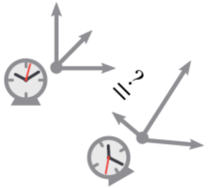
4. account for perturbation in phase

$$\varphi = \varphi_0 + \varphi^{(1)}_{\mathrm{pert}} + \cdots$$

Testing the Einstein Equivalence Principle

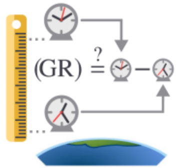


Local Lorentz Invariance



Universality of Free Fall

Universality of the Gravitational Redshift



Universality of Clock Rates

Phys. Rev. D **107**, 064007
PRX Quantum **2**, 040333

Standard-Model Extension

- including light, scalar dilaton field $\varrho(z,t)$ Nucl. Phys. B **423**, 532
  - dark matter (background fields)
  - consistent violation of equivalence principle (sourced by mass)

- Lagrangian including all sectors

Phys. Rev. D **82**, 084033

$$\mathcal{L} = \mathcal{L}_{\text{EM}} + \mathcal{L}_{\text{mat}} + \mathcal{L}_{\text{EH}}$$

electromagnetic matter Einstein-Hilbert

- electromagnetic sector

determinant of metric

$$\mathcal{L}_{\text{EM}} = -\frac{\sqrt{-\mathfrak{g}}}{4\mu_0}F_{\mu\nu}F^{\mu\nu} \mapsto -\frac{\sqrt{-\mathfrak{g}}}{4\mu_0}(1-\varrho\, d_e)F_{\mu\nu}F^{\mu\nu}$$

magnetic susceptibility field strength linear dilaton field coupling constant

$\Rightarrow$ light in gravity and dilaton fields

Phys. Rev. D **105**, 084065

Matter Sector

- leptonic part
  Phys. Rev. D **82**, 084033

leptons Dirac operator masses

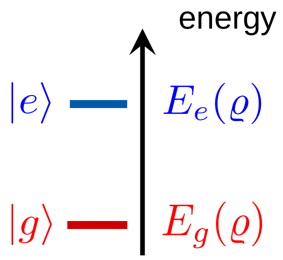
scalar dilaton field coupling constant

$$\sqrt{-\mathfrak{g}}\sum_{i=e,u,d}\bar{\psi}_i\left[i\hbar c\not{D}-m_i c^2\right]\psi_i \mapsto \sqrt{-\mathfrak{g}}\sum_{i=e,u,d}\bar{\psi}_i\left[i\hbar c\not{D}-\underbrace{[1+\varrho\,\tilde{d}_{m_i}]m_i}_{=m_i(\varrho)}\,c^2\right]\psi_i$$

$\Longrightarrow$ lepton mass effectively dilaton dependent

- gauge bosons (photons, gluons, …)

$\Longrightarrow$ couplings effectively dilaton dependent



- masses and internal structure depend on dilaton

effective coupling coefficient

- linear expansion of state-dependent mass

$$m_j(\varrho) \cong m_j[1+\varepsilon_j\,\varrho]$$

Phys. Rev. D **105**, 084065
PRX Quantum **2**, 040333

Metric and Dilaton

- Einstein-Hilbert action

  Phys. Rev. D **82**, 084033

$$\mathcal{L}_{\text{EH}} = \sqrt{-\mathfrak{g}}\,\frac{c^4}{16\pi G}\left[R - \underbrace{2\mathfrak{g}^{\mu\nu}\partial_\mu\varrho\partial_\nu\varrho - \frac{2\varrho^2}{\lambda_\varrho^2}}_{\text{Klein-Gordon type}}\right]$$

Newtonian constant

Ricci scalar

kinetic term

Dilaton Compton wavelength $\lambda_\varrho = \hbar/(cm_\varrho)$

- metric
  - dilaton as perturbation
  - infinite massive plane at $z = 0$:

$$\mathfrak{g}_{\mu\nu} = \eta_{\mu\nu} + \delta^0{}_\mu\delta^0{}_\nu 2\,g_0\,z/c^2$$

Minkowski

gravitational acceleration

- dilaton
  - wave equation

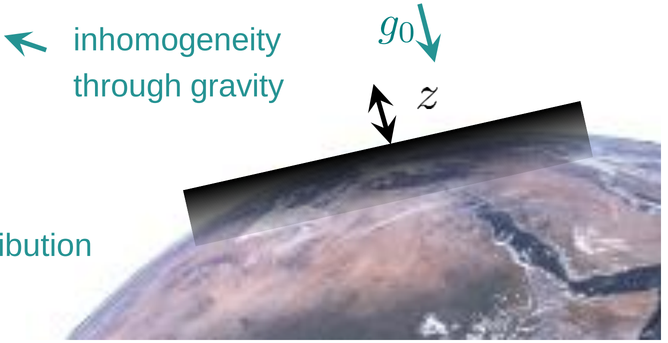
$$\left(\partial_0^2 - \partial_j^2\right)\varrho = \frac{\varrho}{\lambda_\varrho^2} + \frac{g_0}{c^2}\left(2z\partial_j^2 + \partial_z\right)\varrho$$

inhomogeneity through gravity

  - solution

    Phys. Rev. D **105**, 084065

$$\varrho(z,t) \cong \underbrace{\varrho_0\cos\left(\omega_\varrho t - k_\varrho z + \phi_\varrho\right)}_{\substack{\text{oscillating background}\\ \Downarrow \\ \text{dark matter}}} + \underbrace{\varepsilon_S\,g_0\,z/c^2}_{\substack{\text{linear gravitation contribution}\\ \Downarrow \\ \text{EP violation}}}$$



Coupling of Atoms to Dark Matter

- Hamiltonian

$$\hat{H}(\varrho) = \sum_{j=g,e}\left[m_j(\varrho)c^2 + \frac{\hat{p}^2}{2m_j(\varrho)} + m_j(\varrho)g(t)\hat{z}\right] \otimes |j\rangle\langle j|$$

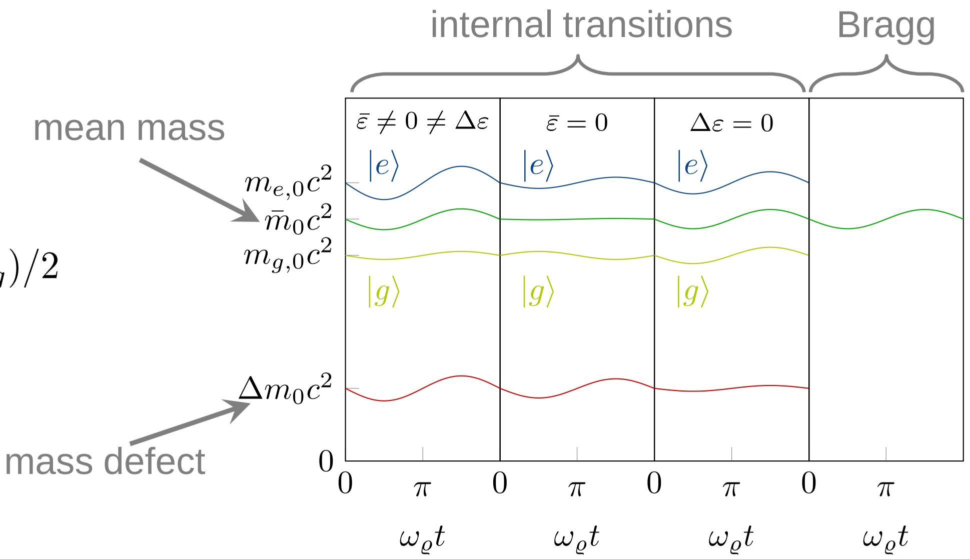
Phys. Rev. A **106**, 032803

$$m_j(\varrho) \cong m_{j,0}\,(1+\varepsilon_j\,\varrho)$$

$$g(t) = g_0[1+\varepsilon_S\,\varrho_0\cos(\omega_\varrho t + \phi_\varrho + \phi_S)]$$

Phys. Rev. Lett. **117**, 261301

- oscillation of atomic energy structure
    - mean coupling $\bar{\varepsilon} = (\varepsilon_e + \varepsilon_g)/2$
    - differential coupling $\Delta\varepsilon = \varepsilon_e - \varepsilon_g$



mean mass

mass defect

AVS Quantum Sci. **5**, 044404

Mass and Gravity Induced Perturbations

- effective replacement

$\lambda_{e/g} = \pm 1$

change of transition freq.

$$m_j(\varrho) \mapsto m_j(t) = \bar{m}_0 \left(1 + \bar{\mu}_{\mathrm{DM}}(t) + \lambda_j \frac{\Delta\mu_0 + \Delta\mu_{\mathrm{DM}}(t)}{2}\right)$$

mean-mass osci.

mass defect
↔ transition freq.

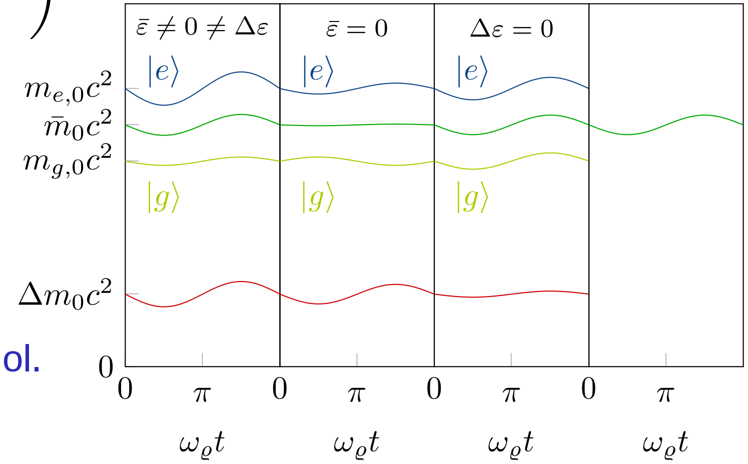
$$g(t) \mapsto g_j(t) = g_0 \left(1 + \bar{\gamma}_{\mathrm{EP}} + \bar{\gamma}_{\mathrm{DM}}(t) + \lambda_j \frac{\Delta\gamma_{\mathrm{EP}}}{2}\right)$$

mean EP violation

osci. of gravity

differential EP viol.

AVS Quantum Sci. **5**, 044404



$\Rightarrow$ perturbations of

rest mass

kinetic energy

potential energy

Change of Atomic Transition Frequency

- perturbed transition frequency

$\omega_\varrho t + \phi_\varrho$

$$\Omega + (\omega_c \Delta\varepsilon + \Omega\bar{\varepsilon})\varrho_0 \cos\vartheta$$

transition frequency

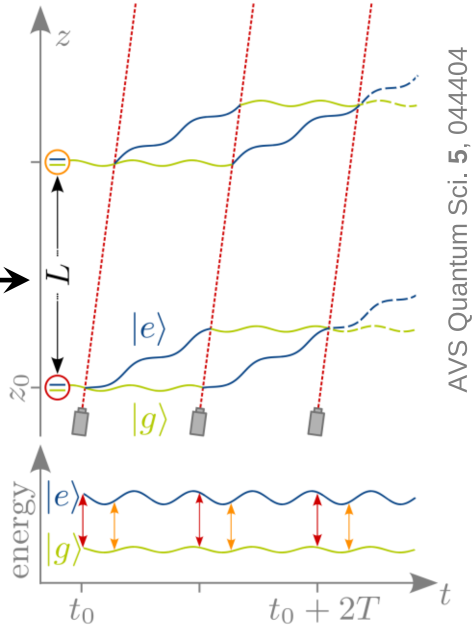
Compton frequency

- (clock) phase contribution

$$\phi_m = -\left[\underbrace{\int_{t_0}^{t_0+T} \mathrm{d}t\,\Omega(t)}_{=\,\Omega T} - \underbrace{\int_{t_0+T}^{t_0+2T} \mathrm{d}t\,\Omega(t)}_{=\,\Omega T}\right]$$

unpert. MZI: $\Rightarrow$ no contribution

$\tau_L = L/c$ →



AVS Quantum Sci. **5**, 044404

Change of Atomic Transition Frequency

- perturbed transition frequency

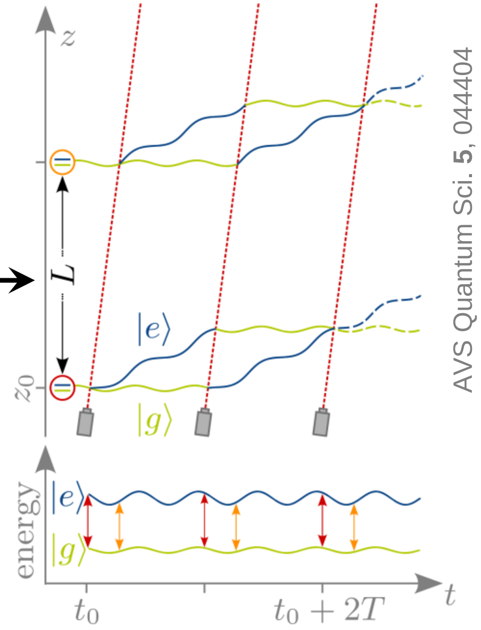
$$\underset{\text{transition frequency}}{\Omega} + (\underset{\text{Compton frequency}}{\omega_c}\Delta\varepsilon + \Omega\bar{\varepsilon})\varrho_0 \cos \overset{\omega_\varrho t + \phi_\varrho}{\vartheta}$$

- (clock) phase contribution

$$\phi_m = -\left[\int_{t_0}^{t_0+T} \mathrm{d}t\,\Omega(t) - \int_{t_0+T}^{t_0+2T} \mathrm{d}t\,\Omega(t)\right]$$

$$= -\varrho_0(\Delta\varepsilon\omega_c + \bar{\varepsilon}\Omega)\underbrace{\left(\int_{t_0}^{t_0+T} \mathrm{d}t\,\cos\vartheta - \int_{t_0+T}^{t_0+2T} \mathrm{d}t\,\cos\vartheta\right)}_{\text{depends on } t_0}$$

$\tau_L = L/c$ →



AVS Quantum Sci. **5**, 044404

- differential setups benefit from laser-phase noise suppression

Phys. Rev. Lett. **110**, 171102

$$\Delta\phi_m = \phi_m(t_0 + \tau_L) - \phi_m(t_0)$$

Clock Contributions in Signal Amplitude

- dilaton phase ϕ_ϱ unknown

  $\Rightarrow$ average squared phase difference

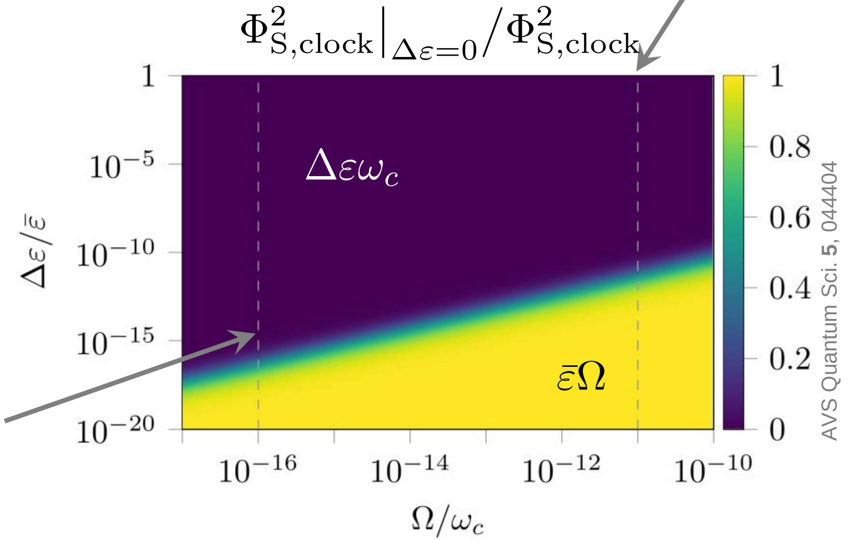
$$\Phi^2_{\mathrm{S,clock}} = 2\int_0^{2\pi} \frac{\mathrm{d}\phi_\varrho}{2\pi}[\Delta\phi_m(\phi_\varrho)]^2$$

Phys. Rev. D **97**, 075020

- clock contribution dominant for single-photon type interferometers

$$\Phi^2_{\mathrm{S,clock}} = 64\frac{(\Delta\varepsilon\omega_c + \bar{\varepsilon}\Omega)^2}{\omega_\varrho^2}\varrho_0^2\underbrace{\mathcal{S}^2(\tau_L)\mathcal{S}^4(T)}_{\text{interferometric factor}}$$

- choose limiting case in accordance with aimed parameter accuracy



clock trans. in Sr

HFS trans. Rb (Raman)

AVS Quantum Sci. **5**, 044404

Internal Transitions and Kinetic Contributions

- hyperfine transitions vs. Compton / clock transitions, e.g. ZAIGA Int. J. Mod. Phys. D **29**, 1940005

$$\left.\frac{\Phi_\mathrm{S}^2}{\Phi_{\mathrm{S,clock}}^2}\right|_{\Delta\varepsilon=0} \cong 1 - \underbrace{\frac{\omega_k}{\Omega}(1+4\bar{\wp}_0)} + \underbrace{2\frac{k g_0 T}{\Omega}} - \underbrace{2\frac{k g_0}{\Omega\omega_\varrho}\frac{\varepsilon_S}{\bar{\varepsilon}}\sin\phi_S}$$

$$\Phi_\mathrm{S}^2 = 2\int_0^{2\pi}\frac{\mathrm{d}\phi_\varrho}{2\pi}\sum_{i,j}\Delta\phi_i(\phi_\varrho)\Delta\phi_j(\phi_\varrho)$$

recoil freq. $\omega_k = \hbar k^2/(2\bar{m}_0)$

→ kinetic effect, $\bar{\wp}_0 = (p_1+p_0)/(2\hbar k)$

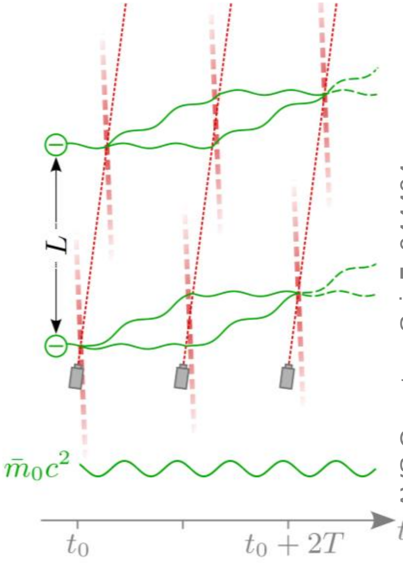
gravity

osci. of gravity

- Bragg-type atom interferometers purely kinetic contributions
  - generally included in previous cases
  - no clear scale, consider zero gravity / horizontal setups cf. MIGA Sci. Rep. **8**, 14064

$$\Phi_\mathrm{S}^2 = 32\frac{\omega_k^2}{\omega_\varrho^2}\bar{\varepsilon}^2\varrho_0^2 \times \text{intf. factor}$$

- due to $\omega_c \gg \Omega \gg \omega_k$ single-photon AIs favourable



AVS Quantum Sci. **5**, 044404

Large Scale Quantum Detectors

- increase spatial dimension of atom interferometers and enclosed space-time area

→ better sensitivity

Phys. Rev. Lett. **125**, 191101

- currently many projects under construction, e. g. MAGIS-100, MIGA, AION, ZAIGA

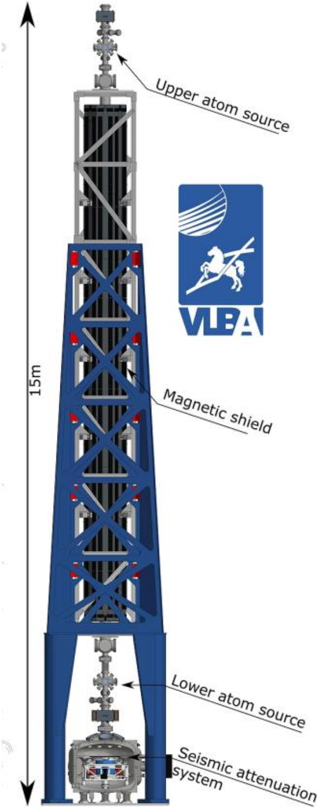
Quantum Sci. Technol. **6,** 044003
Sci. Rep. **8**, 14064
J. Cosmol. Astropart. Phys. **2020**, 011
Int. J. Mod. Phys. D **29**, 1940005

- searches for physics beyond the Standard Model, e. g. ultralight dark matter

AVS Quantum Sci. **5**, 044402

- complementary gravitational-wave detectors

AVS Quantum Sci. **6**, 024701



IQO/LUH

Resonant Mode for Clock Contribution

- inspired by resonant-mode detection for gravitational-wave detection

Phys. Rev. D **94**, 104022
Phys. Rev. D **97**, 075020

- zigzag line → only one upper and one lower path

$$\mathcal{Q}_+(\omega T, Q) = \frac{1}{2}\sin(Q\omega T)\tan\frac{\omega T}{2}$$

$$\Rightarrow \quad \text{resonant mode at} \quad \omega T = \pi$$

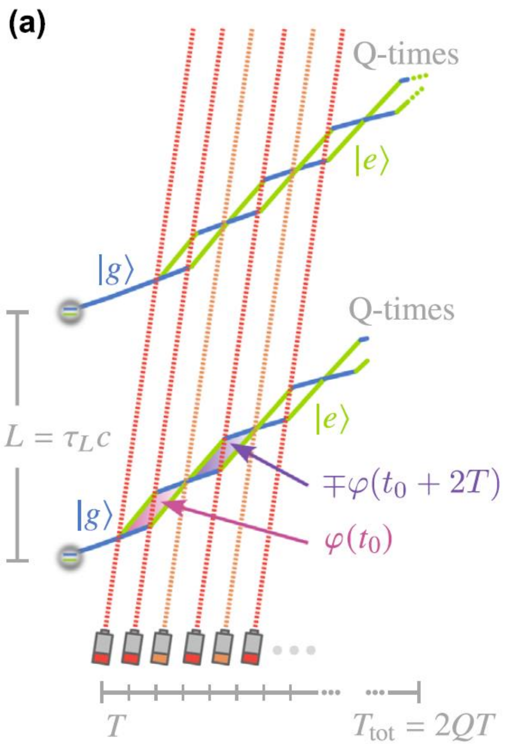
$$|\mathcal{Q}_+(\pi, Q)| = Q$$

- butterfly loops → interchanging upper and lower path

$$\mathcal{Q}_-(\omega T, Q) = \begin{cases} \sin^2\frac{\omega T}{2}\cos(Q\omega T)\,/\cos\omega T & \text{for } Q \text{ odd} \\ \sin^2\frac{\omega T}{2}\sin(Q\omega T)\,/\cos\omega T & \text{for } Q \text{ even} \end{cases}$$

$$\Rightarrow \quad \text{resonant mode at} \quad \omega T = \pi/2$$

$$|\mathcal{Q}_-(\pi/2, Q)| = Q/2$$



AVS Quantum Sci. **6**, 014404

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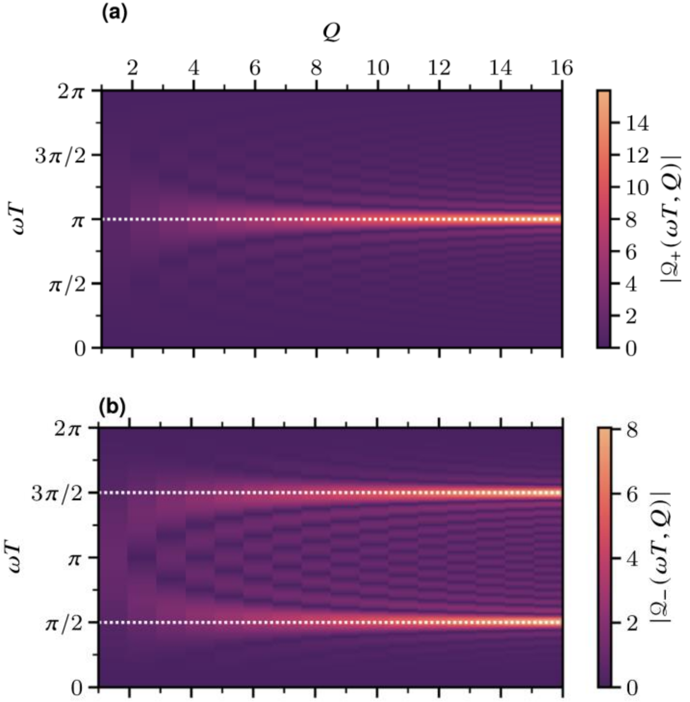


AVS Quantum Sci. **6**, 014404

Baseline Optimisation for Resonant Mode

- minimisation for interferometer height

$\Rightarrow h = B/5$

AVS Quantum Sci. **6**, 014404

- optimal sensitivity to dark matter

local factors

Earth mass

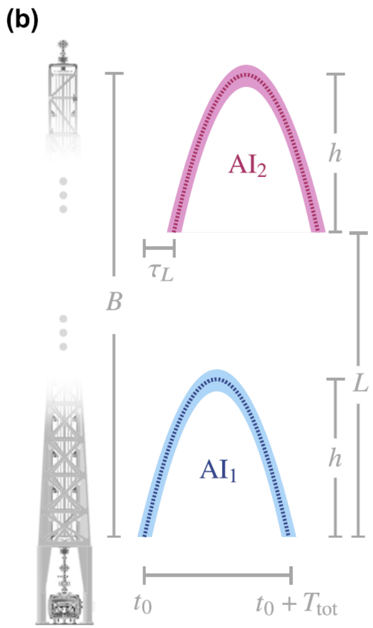
$$\Delta\bar{\varepsilon} = \frac{5}{64}\sqrt{\frac{10\pi}{n_{\mathrm{at}}\nu}}\,\frac{m_{\mathrm{P}}c^2}{\hbar\Omega}\,\frac{L_{\mathrm{P}}}{R_{\mathrm{E}}}\sqrt{\frac{m_{\mathrm{E}}c^2}{\varrho_{\mathrm{DM}}B^3}}$$

transition frequency

Earth radius

probed volume

→ scales with $B^{-3/2}$



AVS Quantum Sci. **6**, 014404

Comparing Gravitational Waves and Dark Matter

Dark matter

- perturbed transition frequency

amplitude DM frequency DM phase

$$\Omega(t) = \Omega + \bar{\varepsilon}\delta\Omega\cos\left(\omega t + \phi\right)$$

unperturbed transition frequency mean coupling parameter

- dominant perturbation from transitions

$$\varphi(t_0) = -\bar{\varepsilon}\delta\Omega\left[\int_{t_0}^{t_0+T}\mathrm{d}t\cos\left(\omega t + \phi\right) - \int_{t_0+T}^{t_0+2T}\mathrm{d}t\cos\left(\omega t + \phi\right)\right]$$

Gravitational waves

- strain induces oscillating laser phase

strain GW frequency GW phase

$$\varphi_\ell(z,t) = k_\ell z h\sin(\omega t + \phi)$$

effective wave vector

Class. Quantum Grav. **38**, 145025

- dominant perturbation from laser phase

$$\varphi(t_0, z) = k_\ell z h\int_{t_0}^{t_0+2T}\mathrm{d}t\sin(\omega t + \phi)$$

$$\times[\delta(t - t_0) - 2\delta(t - T - t_0) + \delta(t - 2T - t_0)]$$

Perspectives

- multi-loop geometries with boundary conditions and large momentum transfer

  Nat. Commun. **12**, 2544

- network schemes

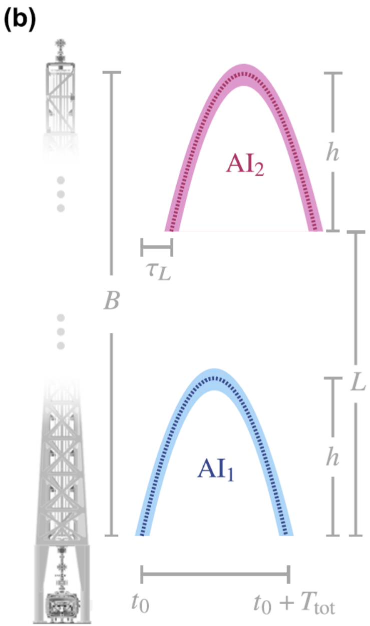
  Quantum Sci. Technol. **6**, 034004

- noise compensation (gravity gradient noise)

  Phys. Rev. D **107**, 055002

- exploit entanglement in atom interferometers

  Phys. Rev. Lett. **127**, 140402



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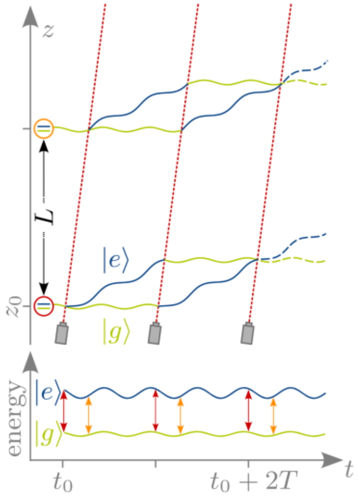

AVS Quantum Sci. **5**, 044404

Thank you for your attention!

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DD and Enno Giese

“Clock Transitions Versus Bragg Diffraction in Atom-interferometric Dark-matter Detection”

AVS Quantum Sci. **5**, 044404 (2023)

F. Di Pumpo, A. Friedrich, E. Giese

“Optimal baseline exploitation in vertical dark-matter detectors based on atom interferometry”

AVS Quantum Sci. **6**, 014404 (2024)

A. Bott, F. Di Pumpo, E. Giese

“Atomic diffraction from single-photon transitions in gravity and Standard-Model extensions”

AVS Quantum Sci. **5**, 04402 (2023)

our team

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Large Scale Quantum Detectors

https://pubs.aip.org/aqs/collection/1079/Large-Scale-Quantum-Detectors

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