Extending the Kibble-Zurek Mechanism to Weakly First-Order Phase Transitions

Fumika Suzuki (with Wojciech H. Zurek)

Topological defect formation in a phase transition with tunable order, Phys. Rev. Lett. 132, 241601 (2024)

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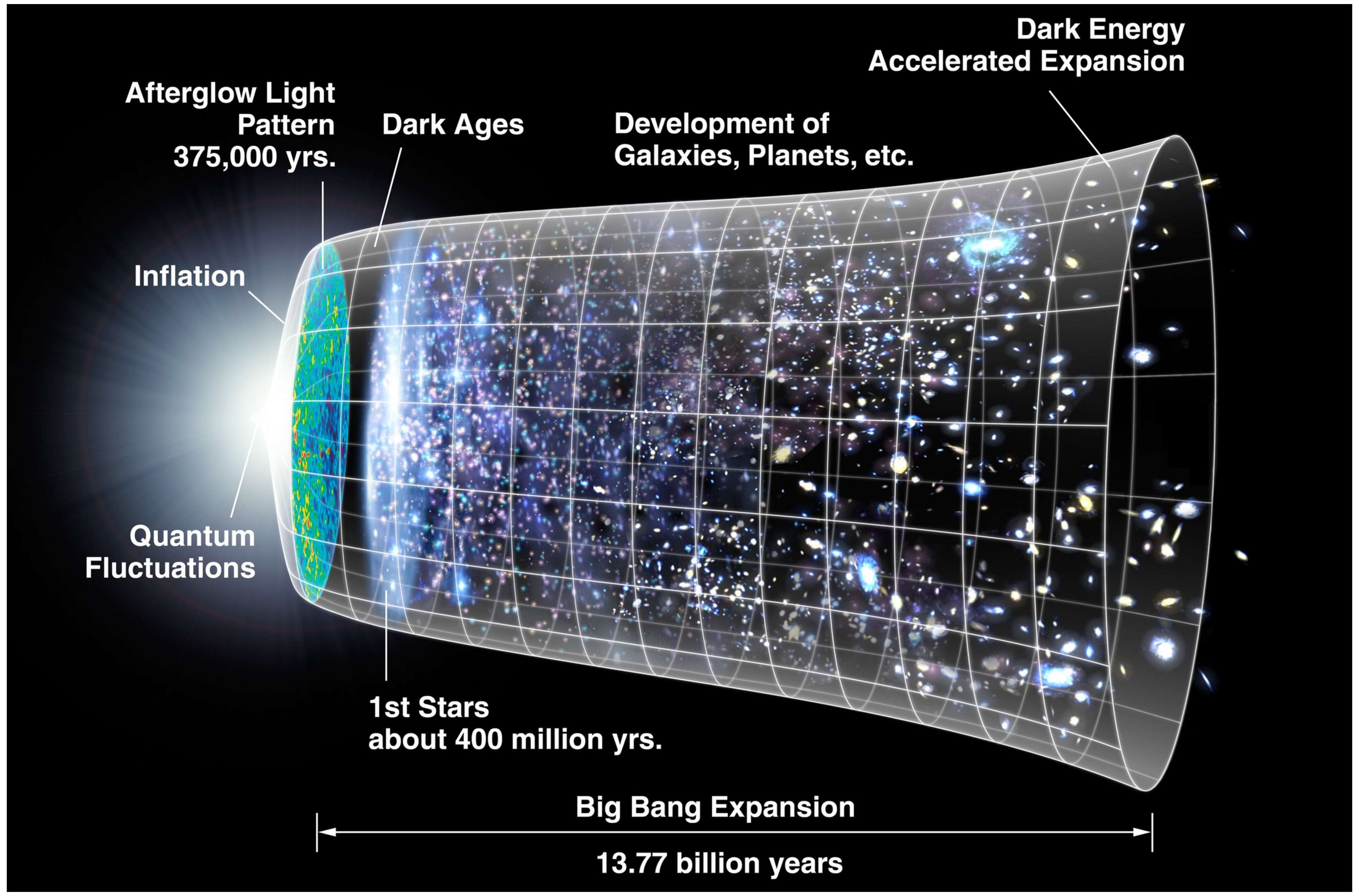
Cosmological phase transition in the early universe

liquid water

solid ice

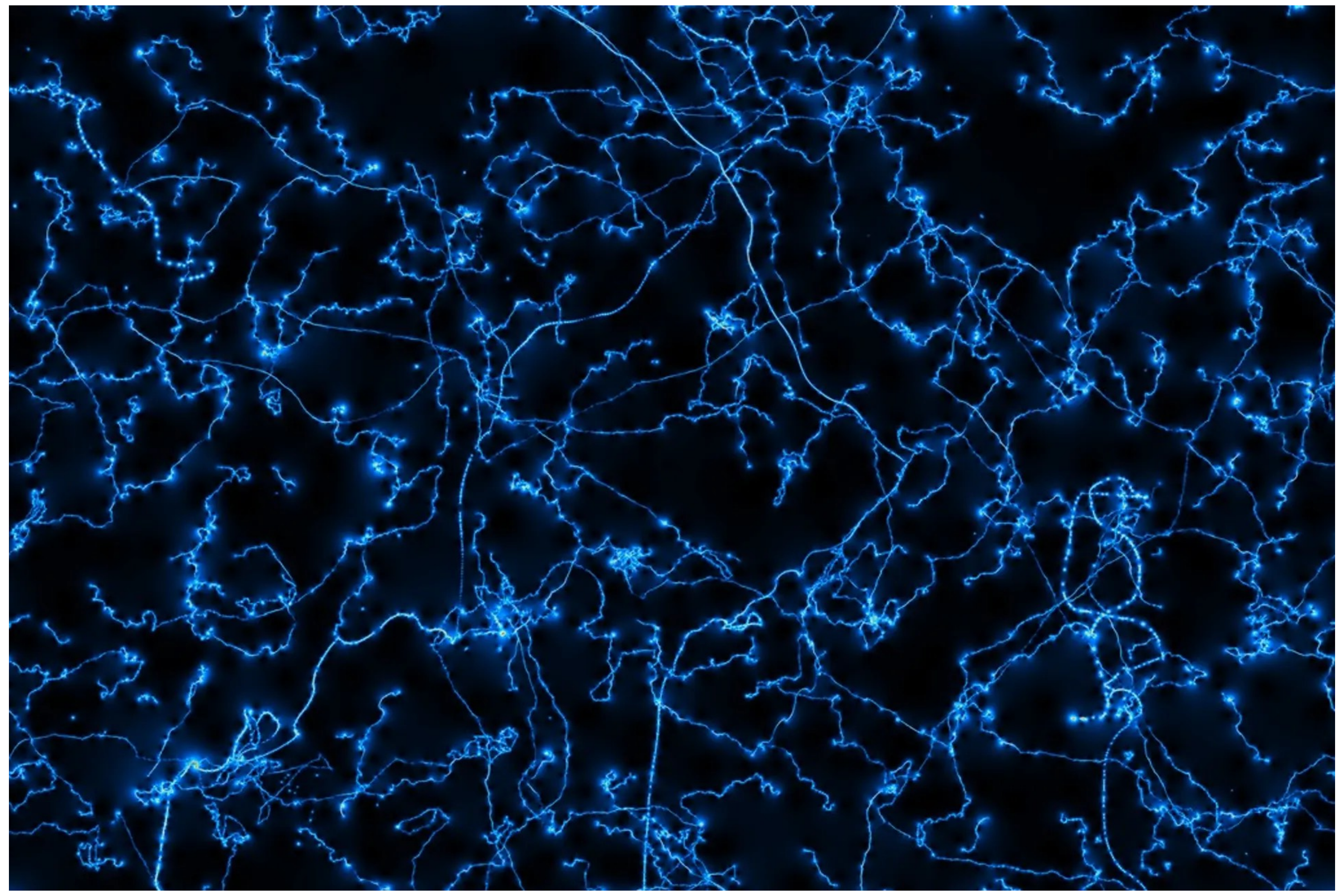
Phase transition of universe after Big Bang as it cooled

Tom Kibble



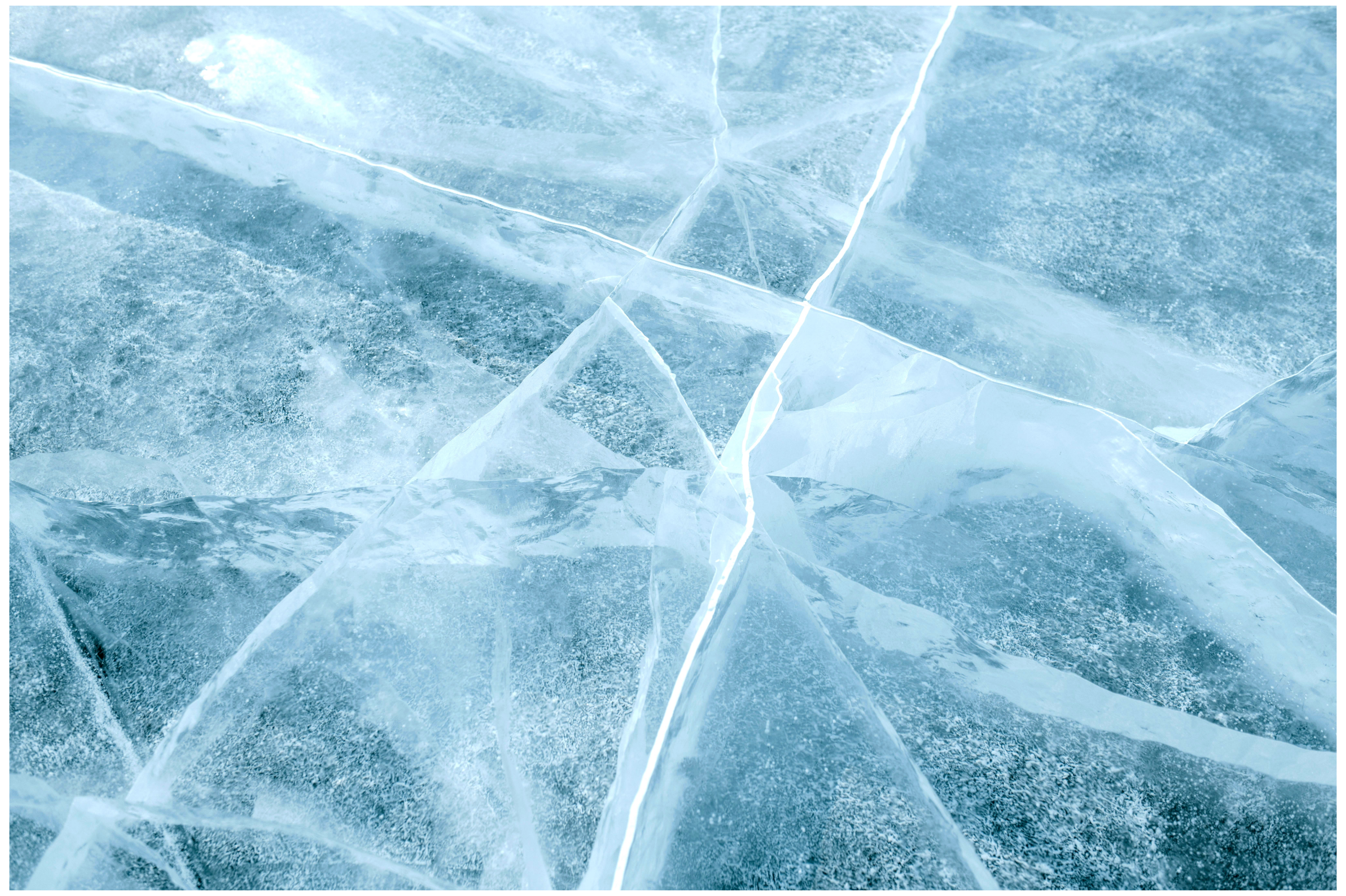
Due to phase transitions, separated regions can have different values of the field
→ topological defects form when different values of the field meet

hot universe



cold universe with defects

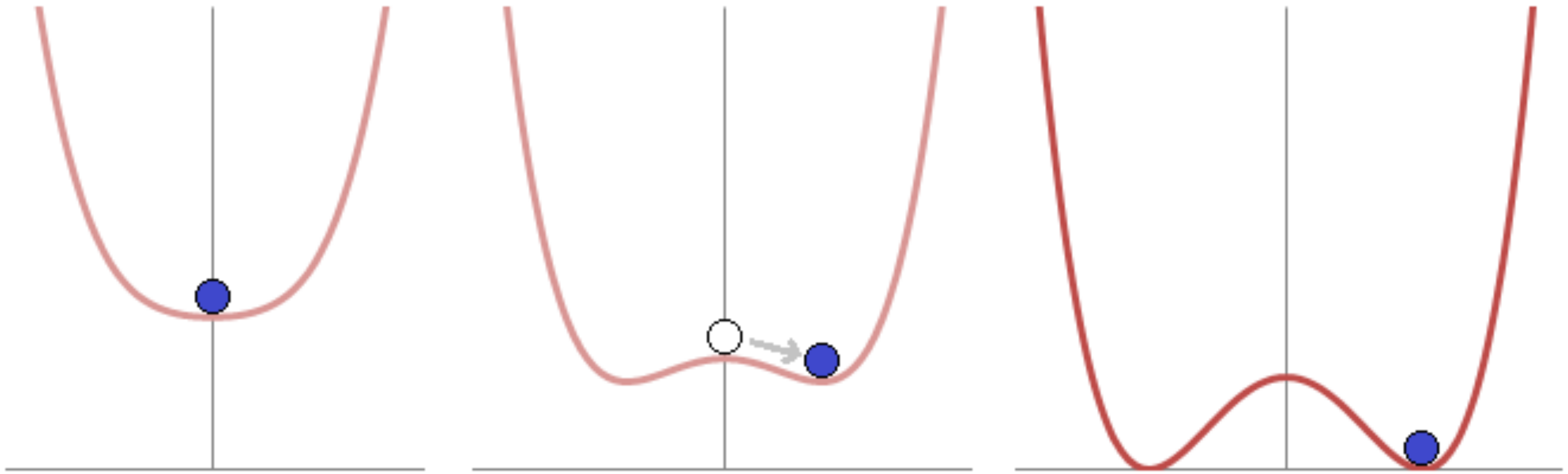
liquid water



solid ice with cracks

Kibble-Zurek mechanism → density of defects

Second-order phase transition

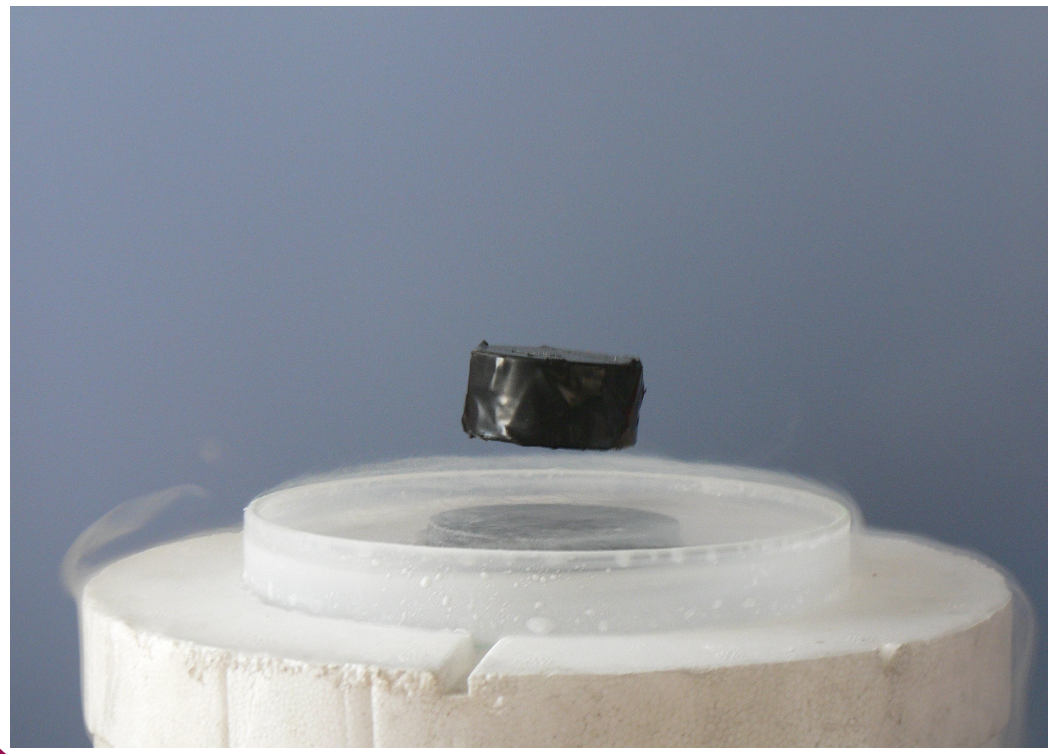


Spontaneous symmetry breaking

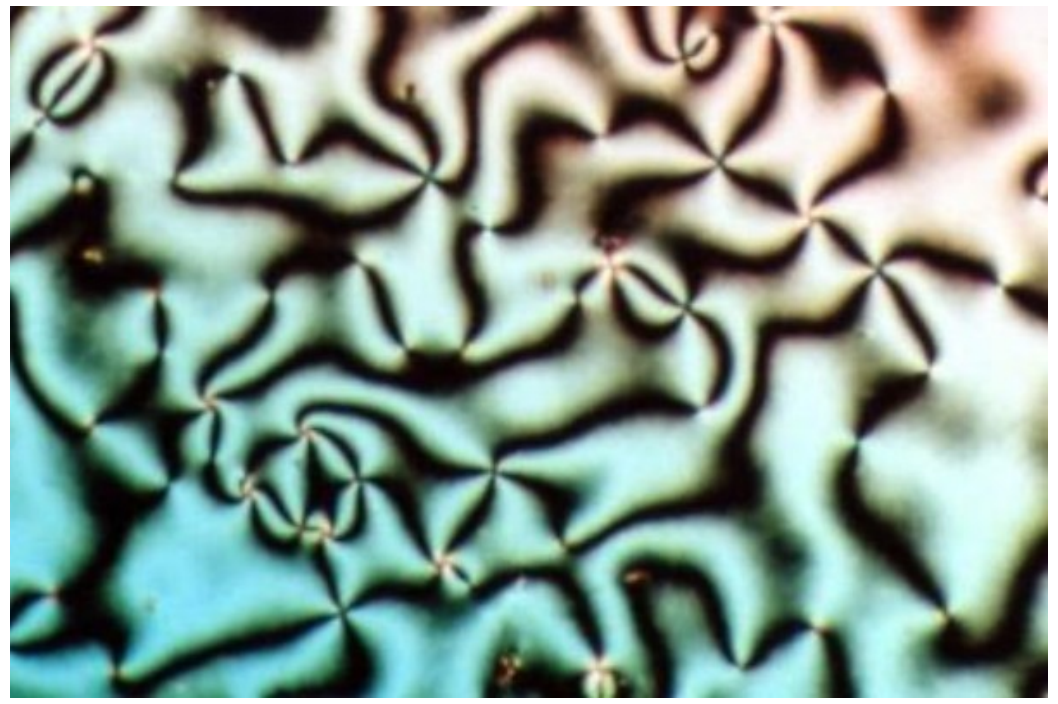
Topological defects form where different broken symmetry choices meet

KZM

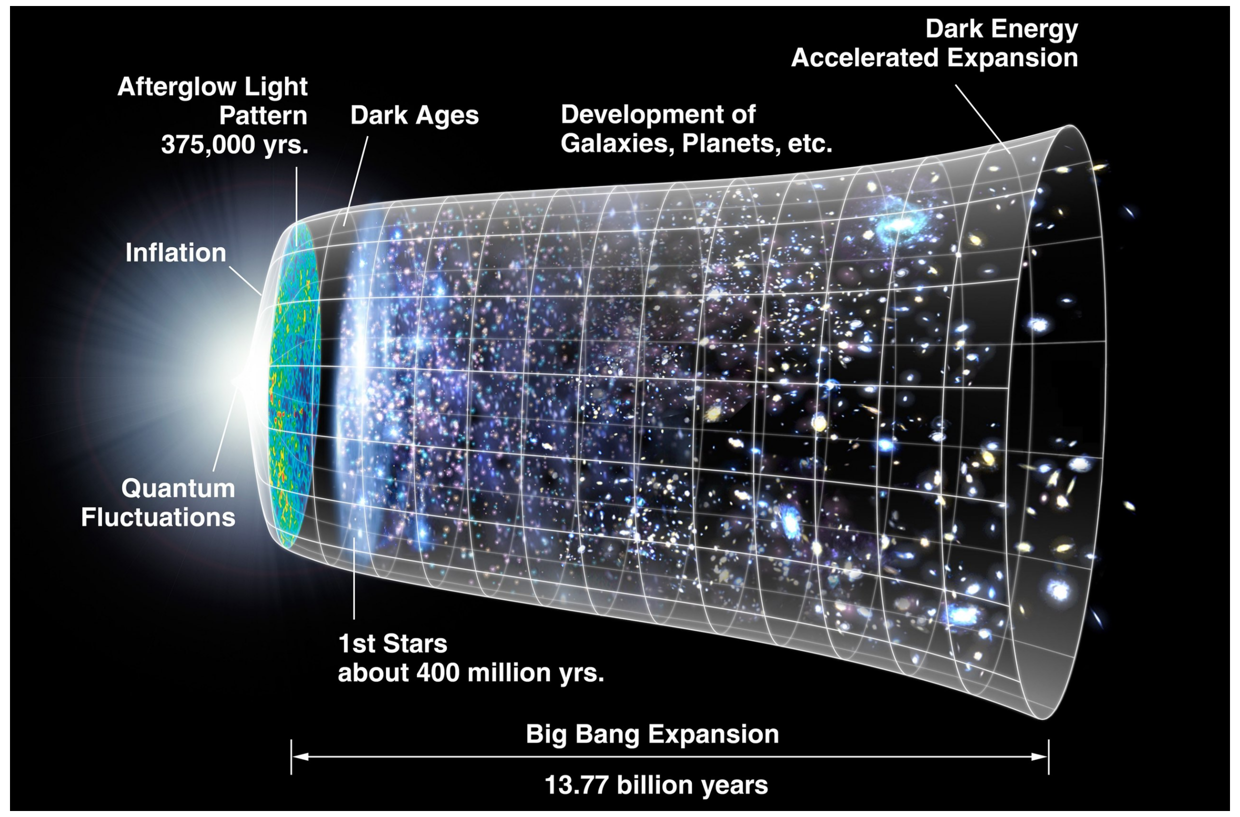
superconductors



liquid crystals



cosmological phase transition



Kibble-Zurek mechanism (KZM)

(Predict defects density by quantity describing equilibrium behavior near the critical point)

Equilibrium correlation length & relaxation time

$$\xi(\varepsilon) = \frac{\xi_0}{|\varepsilon|^{\nu}} \qquad \tau(\varepsilon) = \frac{\tau_0}{|\varepsilon|^{z\nu}}$$

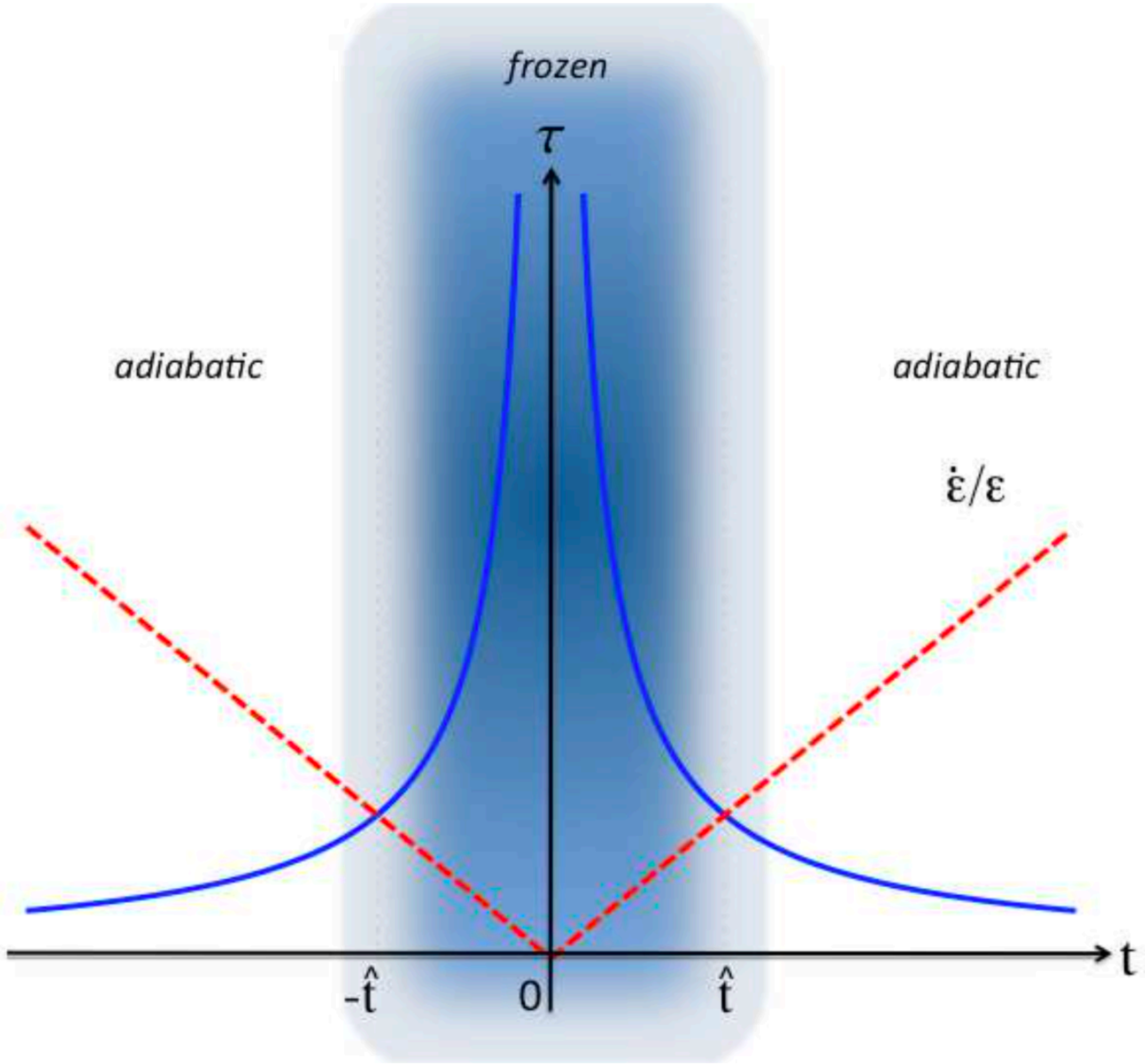
Relative distance to the critical point $\varepsilon = \dfrac{\lambda_c - \lambda}{\lambda_c}$

Linear quench $\varepsilon(t) = \dfrac{t}{\tau_Q}$ ⟵ quench time

Domain size $\hat{\xi} \equiv \xi[\hat{\varepsilon}] = \xi_0 \left(\dfrac{\tau_Q}{\tau_0}\right)^{\frac{\nu}{1+z\nu}}$

Density of topological defects

$$n \sim \frac{\hat{\xi}^d}{\hat{\xi}^D} = \frac{1}{\xi_0{}^{D-d}} \left(\frac{\tau_0}{\tau_Q}\right)^{(D-d)\frac{\nu}{1+z\nu}}$$



D: dimensions of space *d*: dimensions of defects

A. Del Campo & W. H. Zurek, Int. J. Mod. Phys. A **29,** 1430018 (2014)

Numerical test of second-order phase transition

Langevin equation

$$\ddot{\phi} + \eta\dot{\phi} - \partial_{xx}\phi + \partial_\phi V(\phi) = \vartheta$$

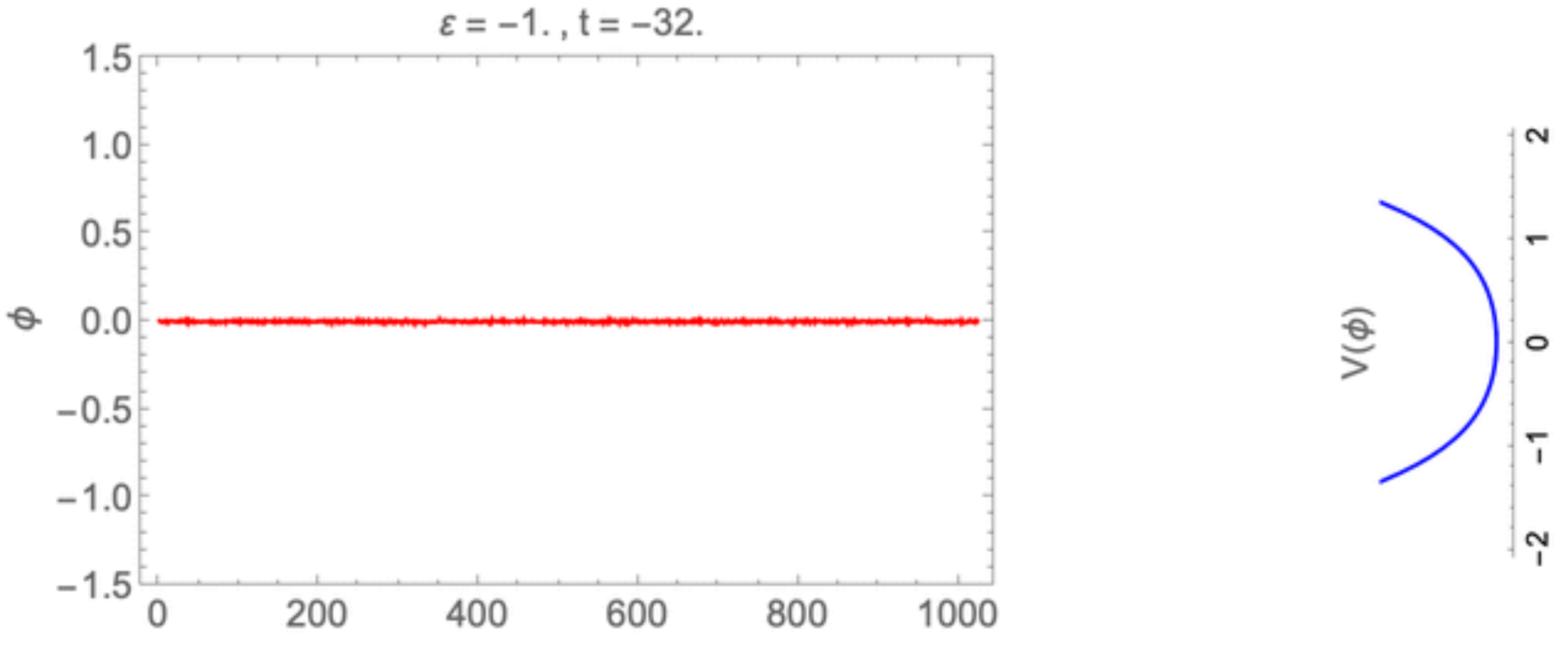
← noise

order parameter

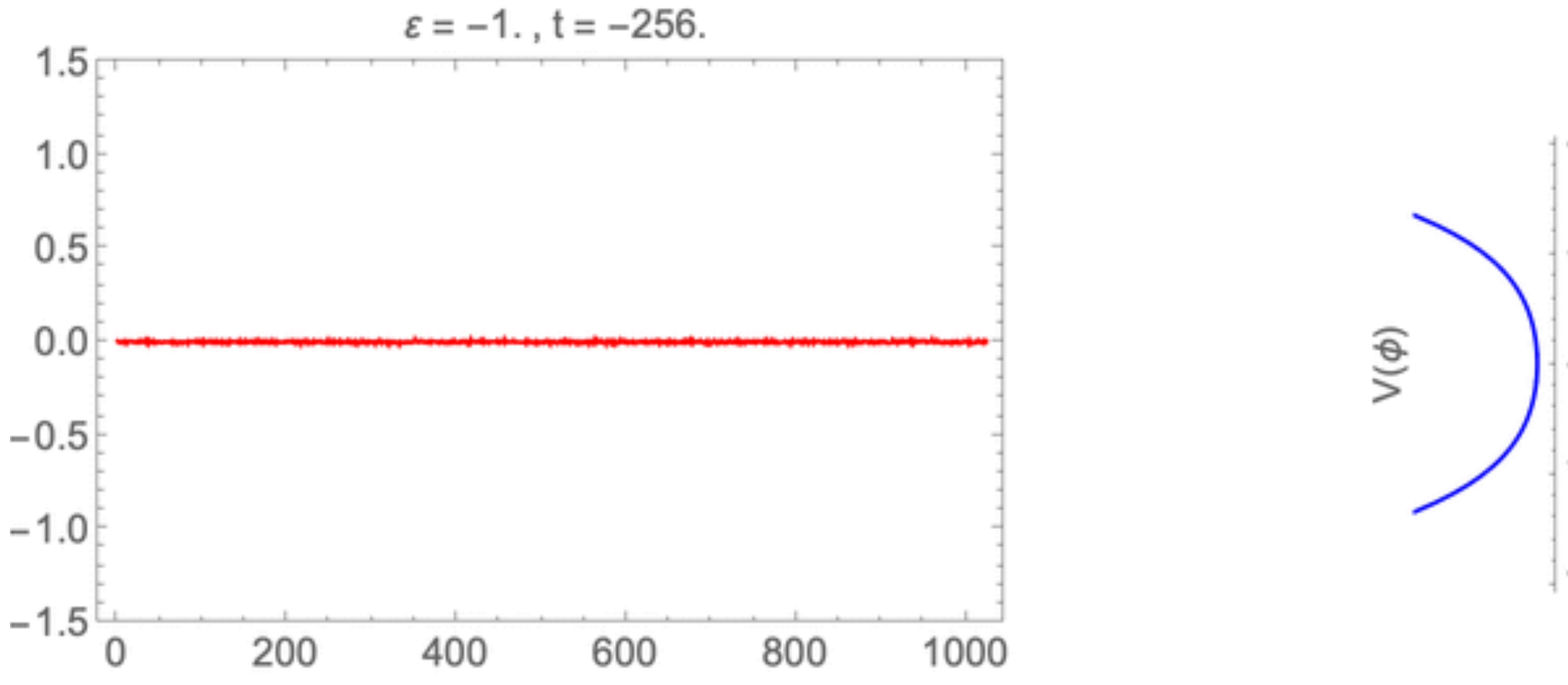
damping constant

$$V(\phi) = (\phi^4 - 2\epsilon\phi^2)/8$$

$$\epsilon = t/\tau_Q$$

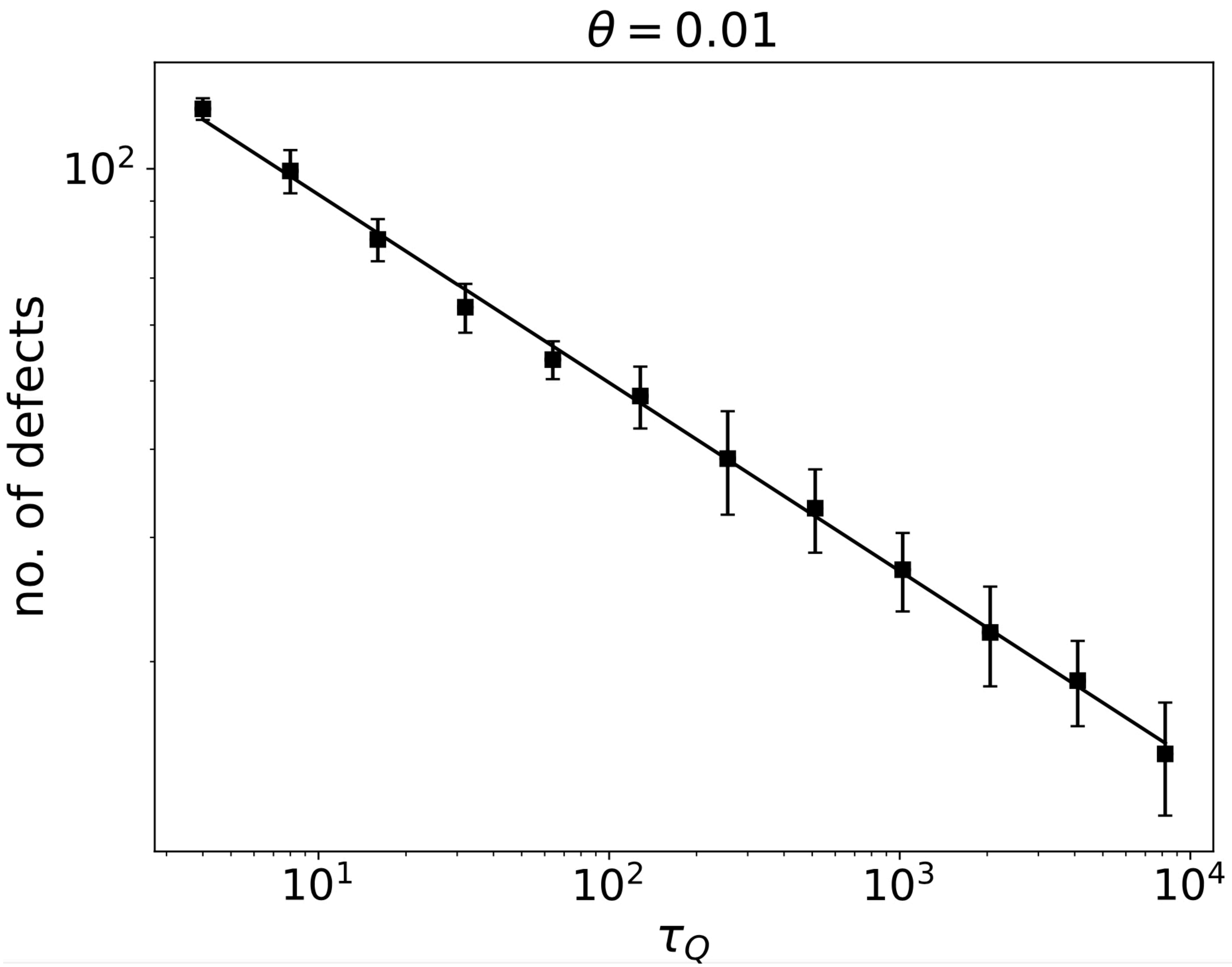


Fast quench (small τ_Q)



Slow quench (large τ_Q)

Number of defects vs τ_Q



Larger τ_Q → slower quench → fewer defects

$$N = N_0 \tau_Q^{-a}$$

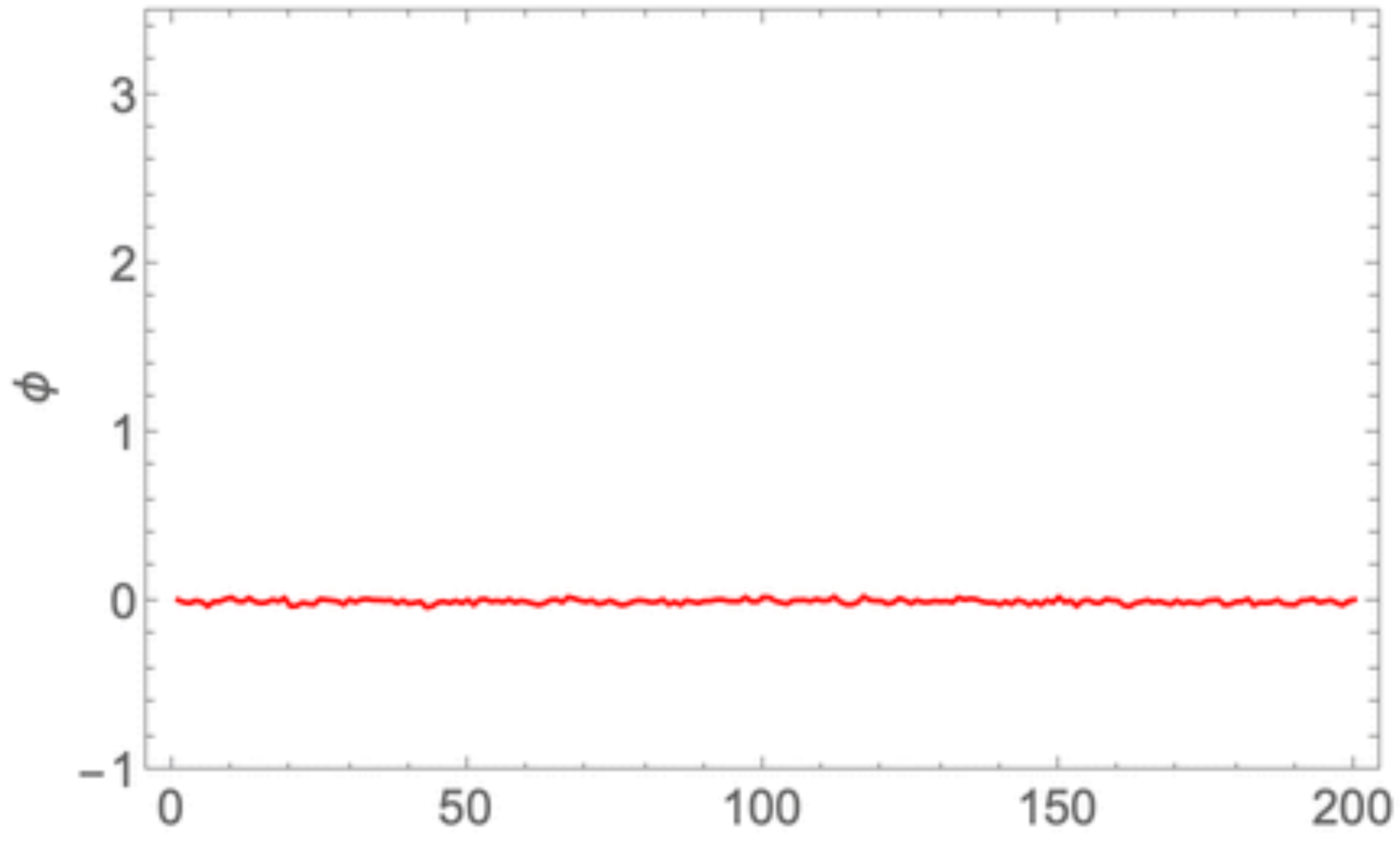
$a = 0.26$ $\qquad$ $N_0 = 164$

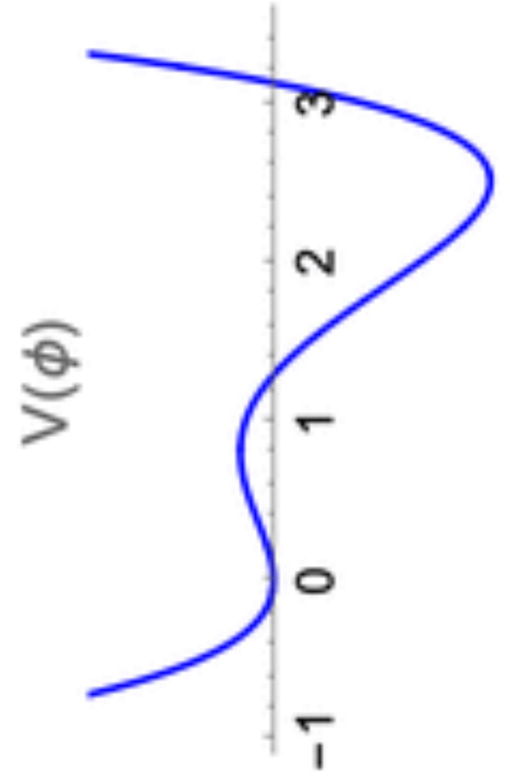
Agrees well with theoretical prediction 1/4

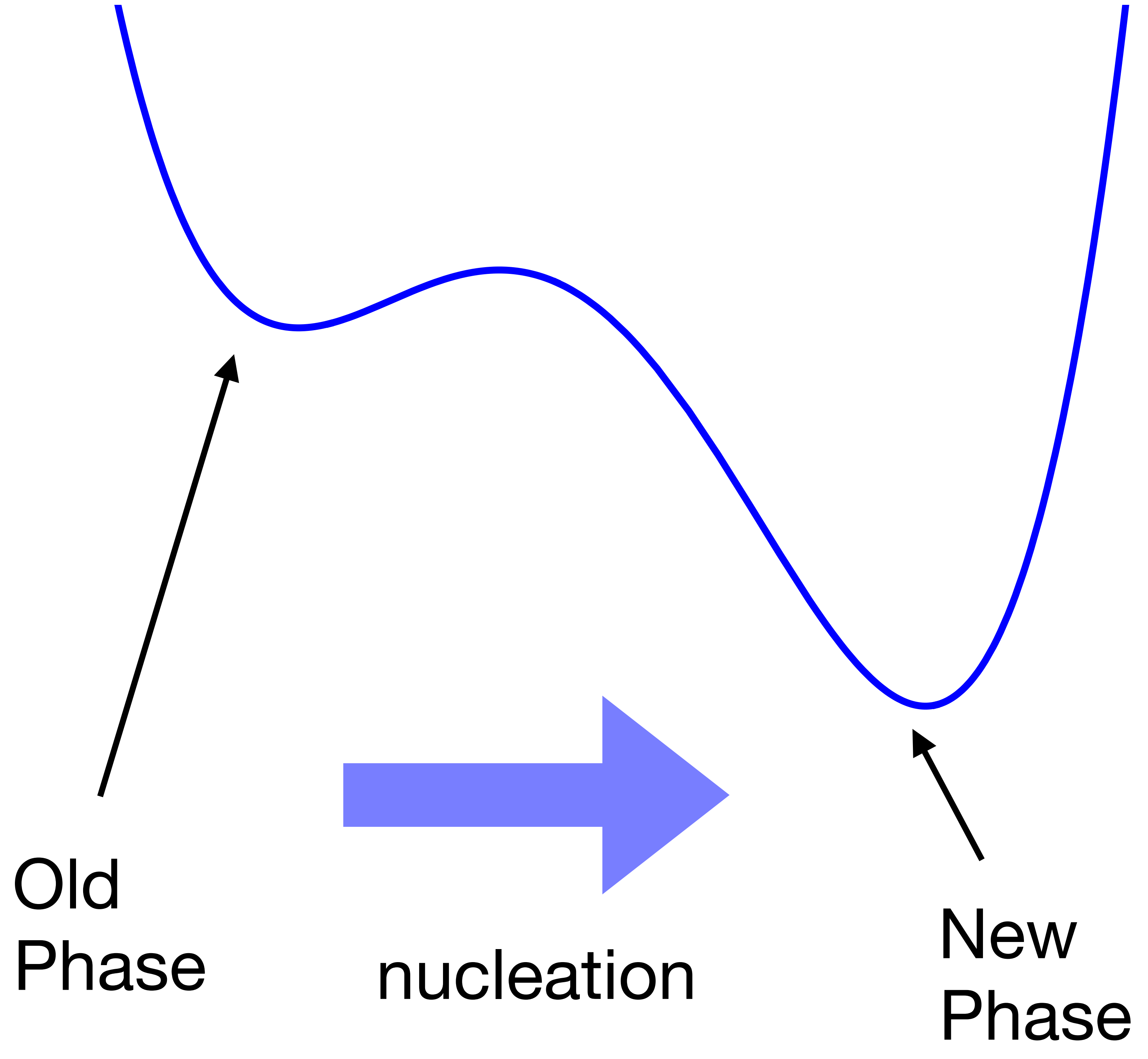
(Power-law decreases of no. of defects as predicted by KZM)

Review: First-order phase transition

e.g., water/ice phase transition, false vacuum decay (cosmology)







Nucleation → nucleus grows → new phase

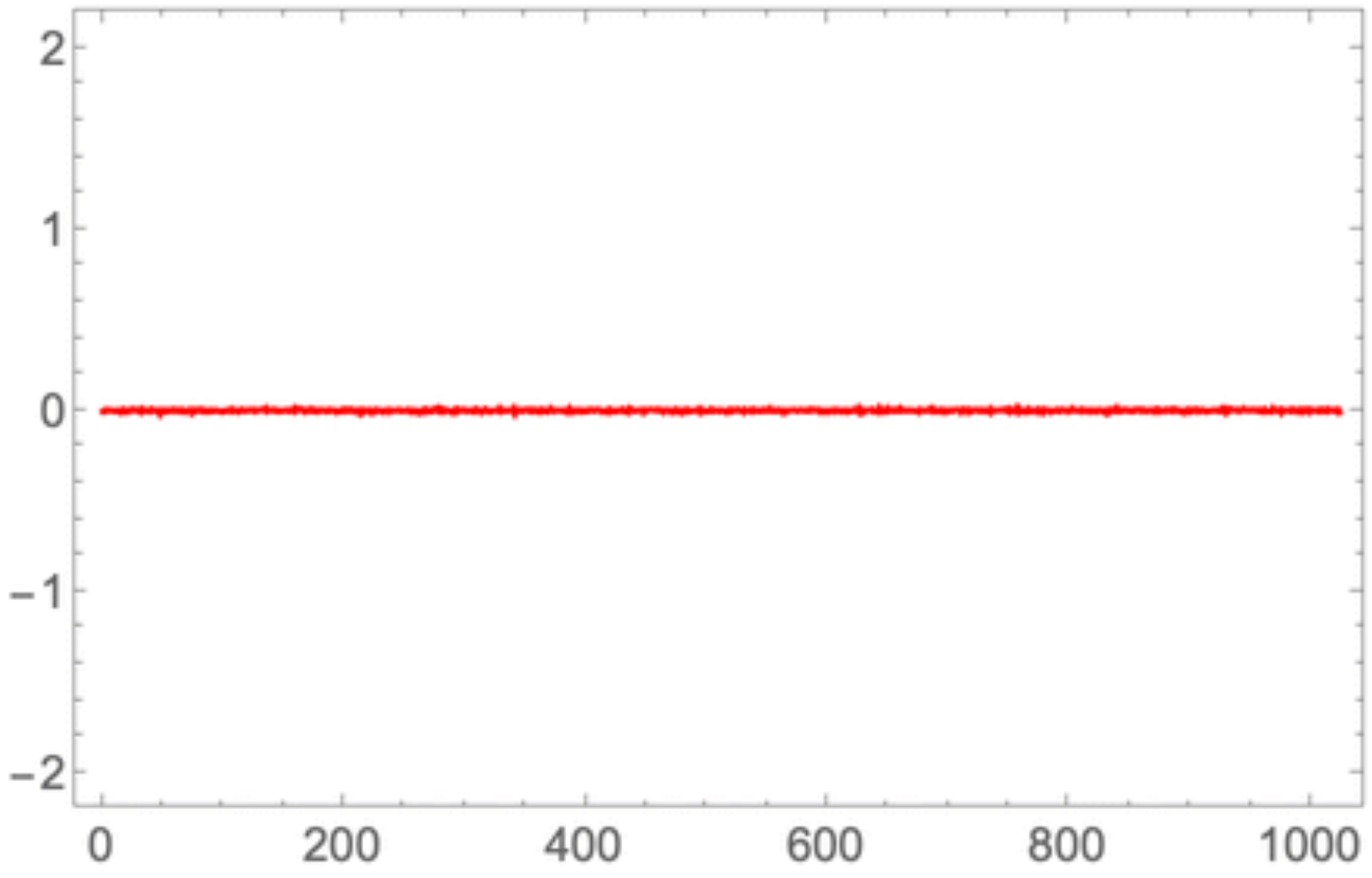
Defects creation in first-order phase transition

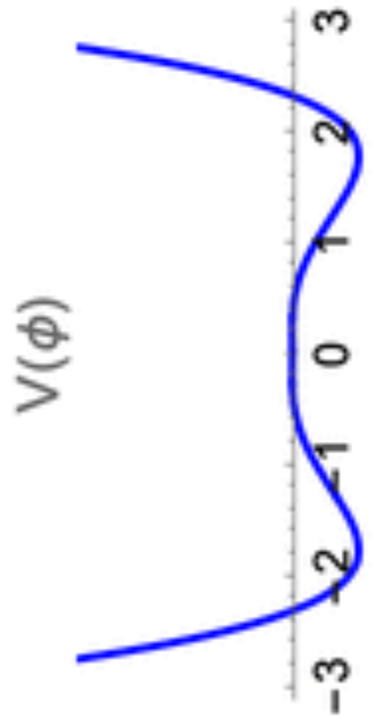
of defects by nucleation

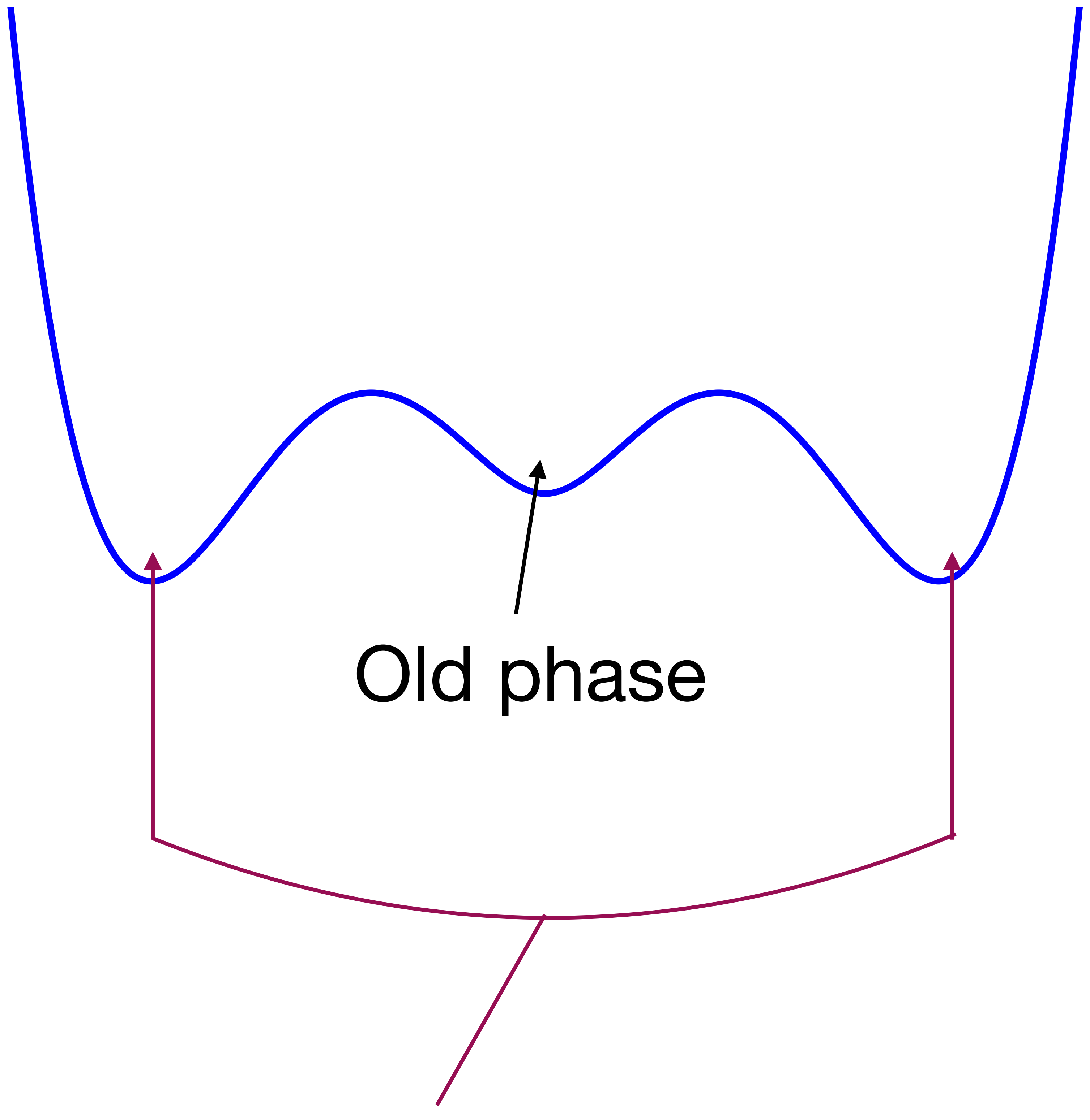
nucleation rate

$$n_{nuc} \propto \Gamma/v$$

nucleus growth velocity

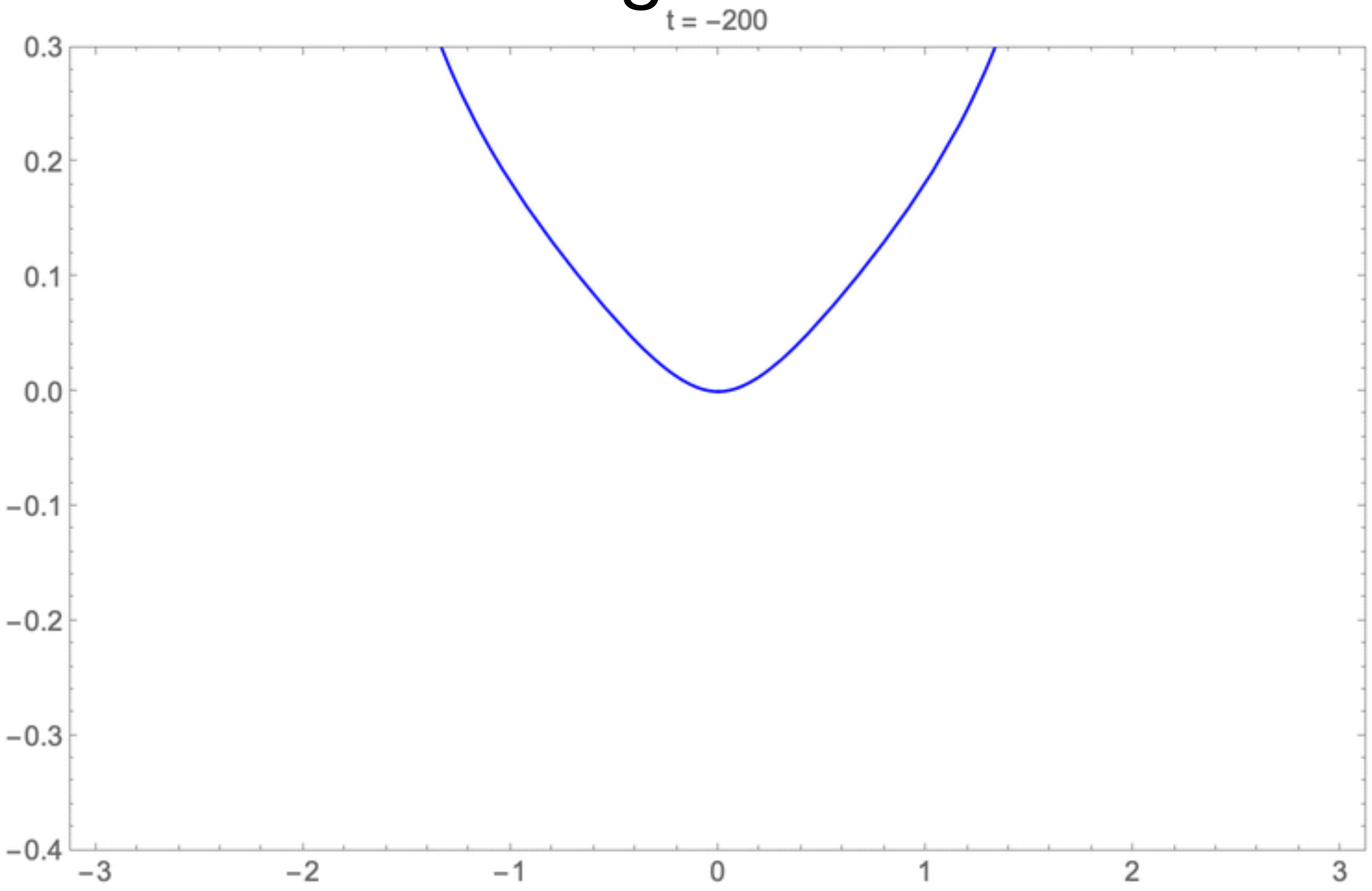






Weakly first-order phase transition

Potential changes with time



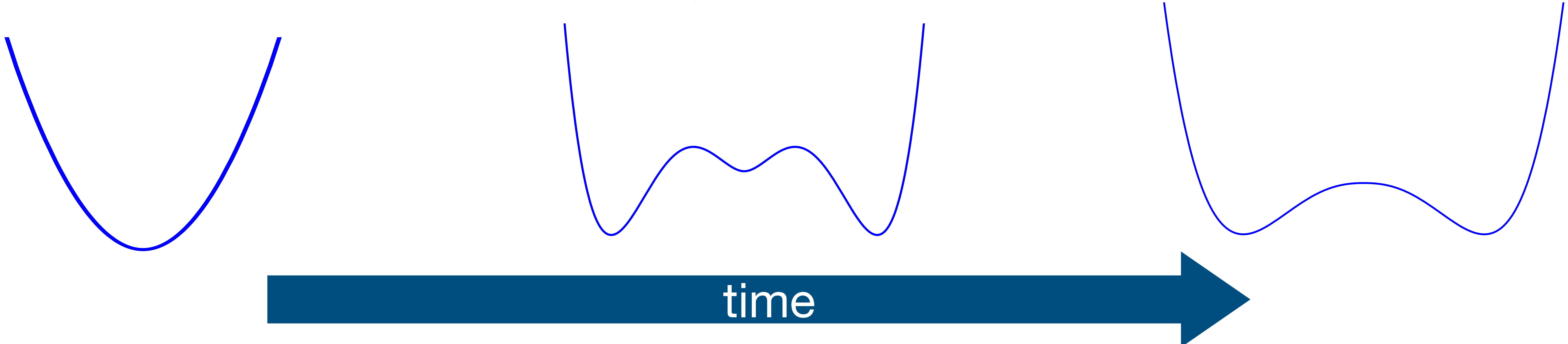
$$V(\phi) = (\phi^4 - 2\epsilon\phi^2)/8 - c|\phi|^3/3$$

1st order

$$\epsilon = t/\tau_Q$$

superconducting phase transition
smectic-*A* to nematic liquid crystal

B. I. Halperin, T. C. Lubensky and Shang-keng Ma,
PRL **32,** 292 (1973)



Topological defects creation in weakly first-order phase transition

Literature: look for KZM-like unified power-law for defects formation

Suzuki, Zurek:
a mixture of KZM & nucleation is what actually happens

# of defects in weakly first-order phase transitions

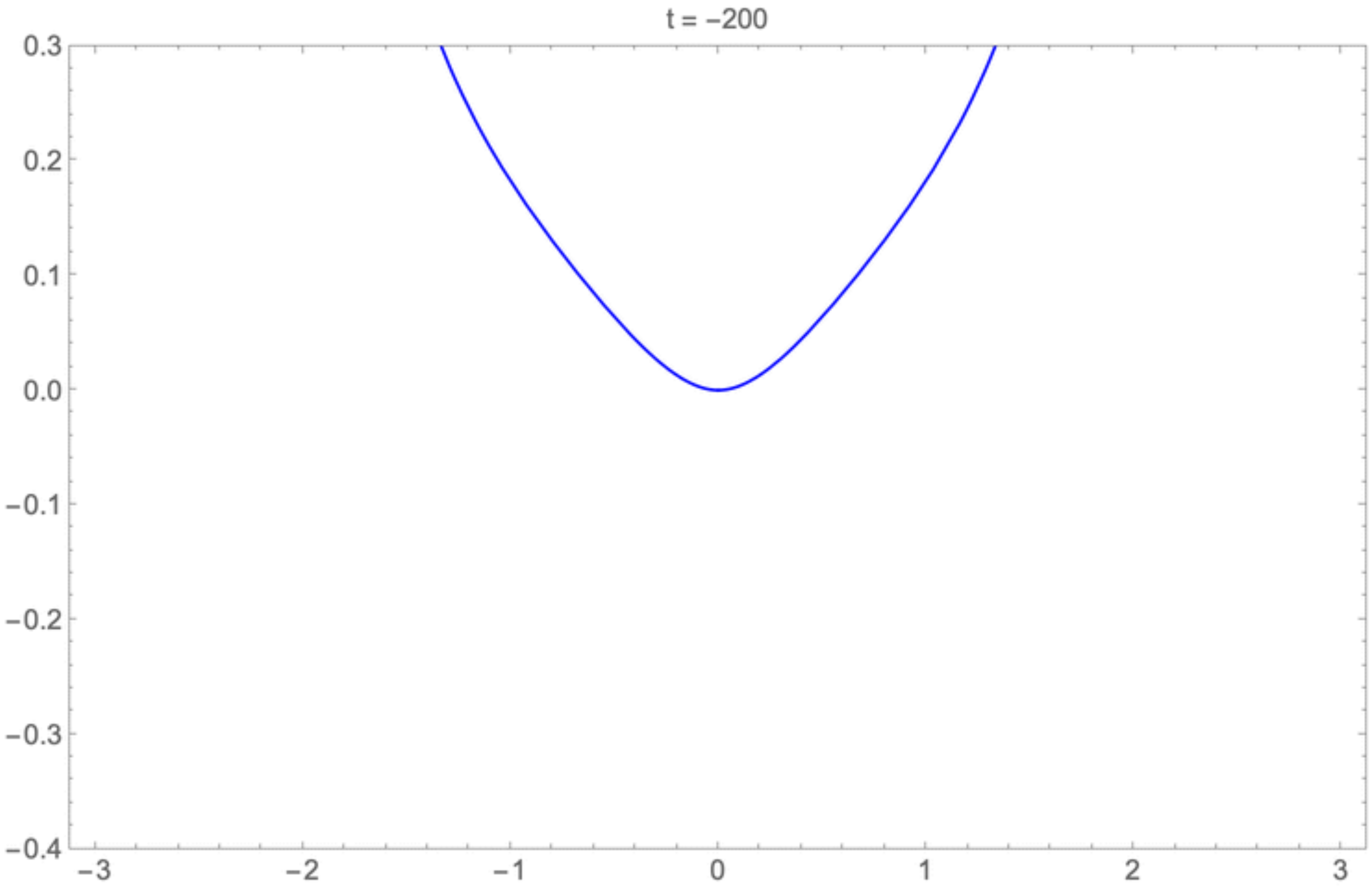
$$n = f n_{nuc} + (1 - f) n_{KZM}$$

# of defects by nucleation
(depends on nucleation rate
nucleus growth velocity)

# of defects by KZM

f: fraction of volume
covered by new phase via nucleation

Potential changes with time



Nucleation can only occur between t_1 and t_2

t_1 : ϕ starts to interact with nucleation barriers

$$\sqrt{\langle \phi^2 \rangle} \sim \phi_{\text{barrier}}$$

(Location of barrier)

t_2 : temperature θ becomes larger than B (energy of the barrier)

Volume fraction covered by new phase due to nucleation

Avrami equation: $$f = 1 - \exp(-\Omega)$$

where $$\Omega = \int_{t_1}^{t_2} \Gamma(\epsilon(t))\mathcal{V}(t,t_2)dt$$

$\mathcal{V}(t,t_2) = \sigma \int_t^{t_2} v(\tau)d\tau$ ↔ volume of nucleus at t_2 which was formed at t

Nucleus shape constant (σ)

Nucleus growth velocity ($v(\tau)$)

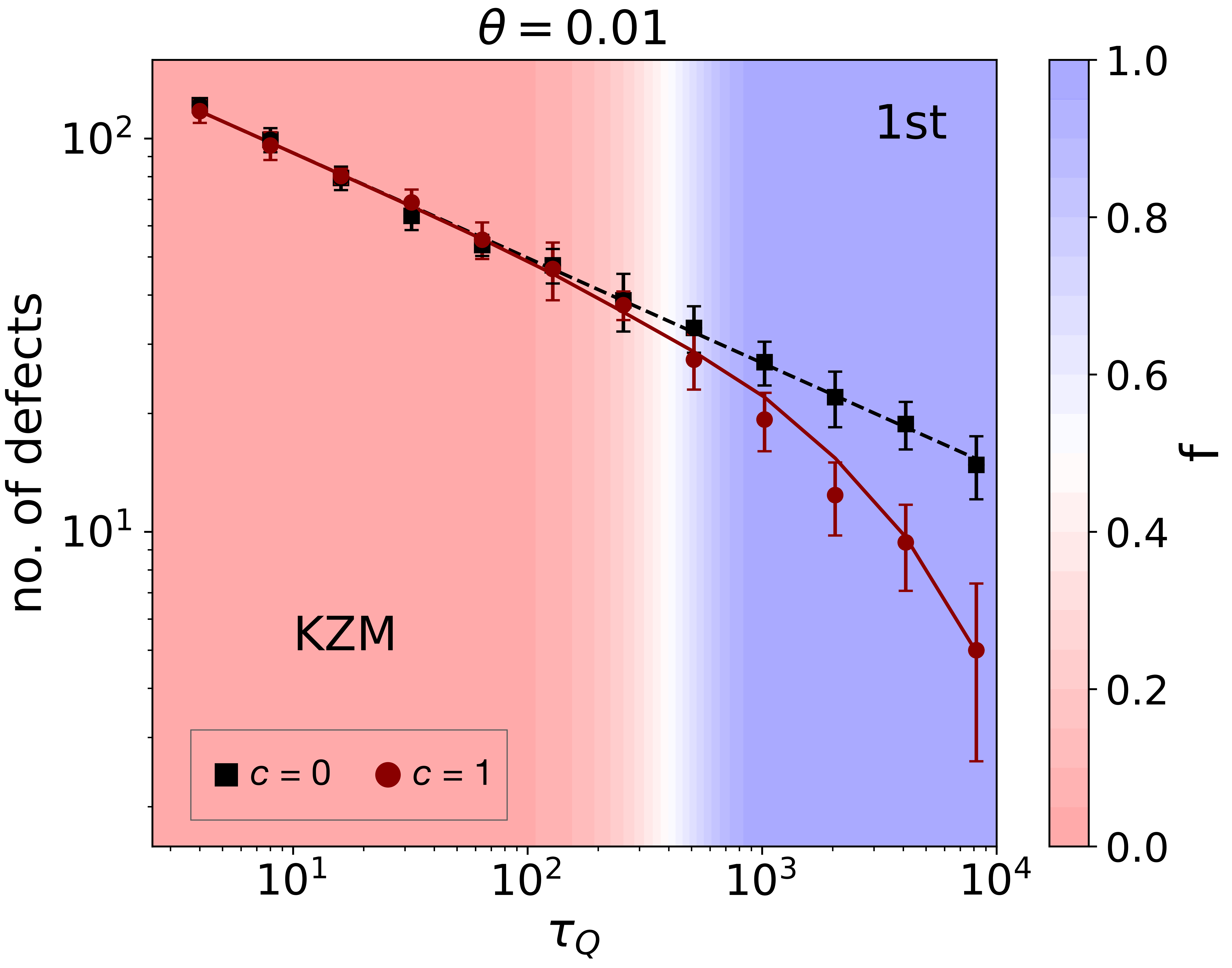
f=1 ↔ all space is covered by new phase due to first-order phase transition (nucleation) before second-order phase transitions

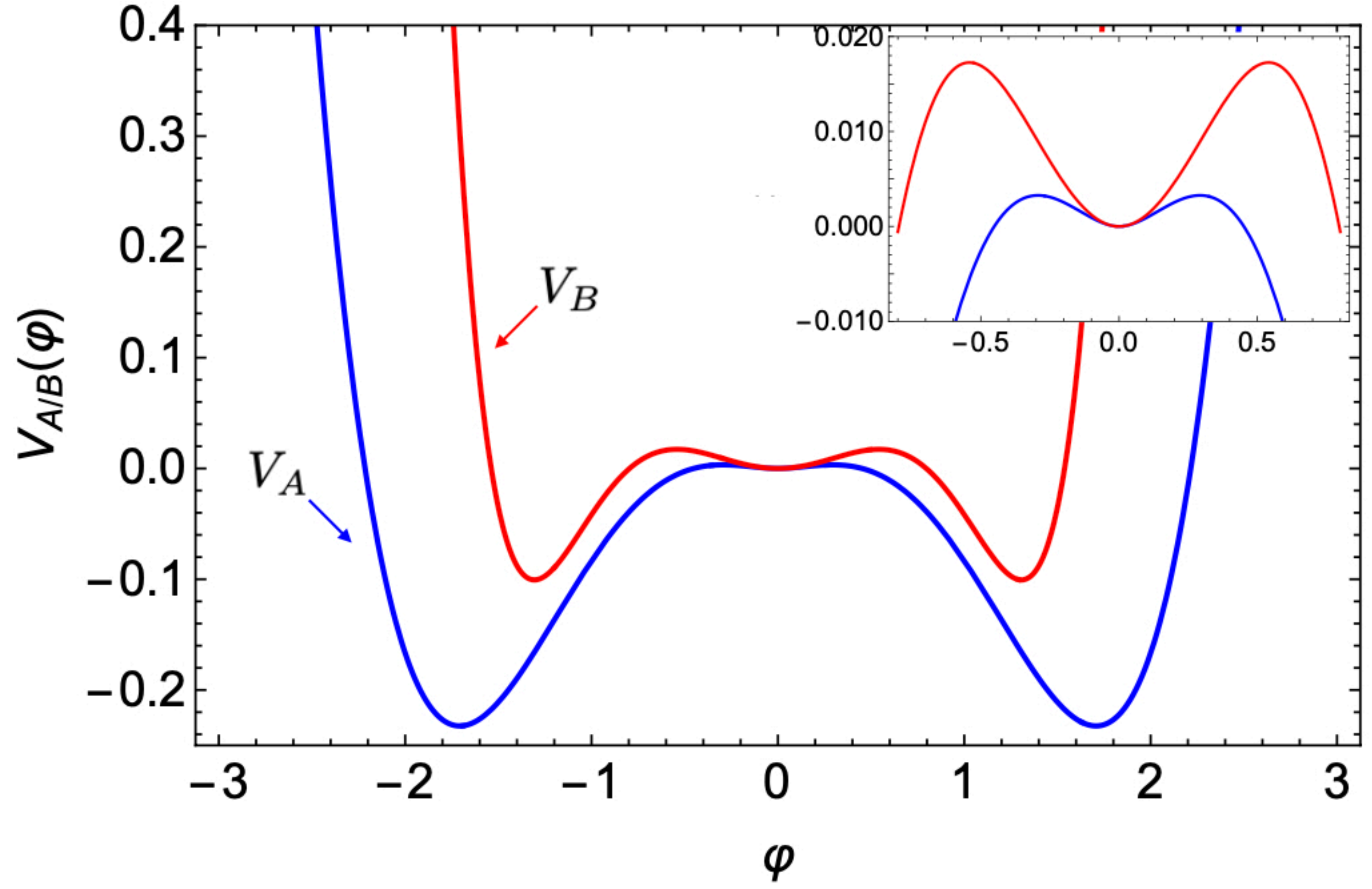
Number of defects in weakly first-order phase transition

$$n = f n_{nuc} + (1 - f) n_{KZM}$$

$$n_{nuc} \propto \Gamma / v$$

$$n_{KZM} \propto \tau_Q^{-a} \text{ with } a = 1/4$$





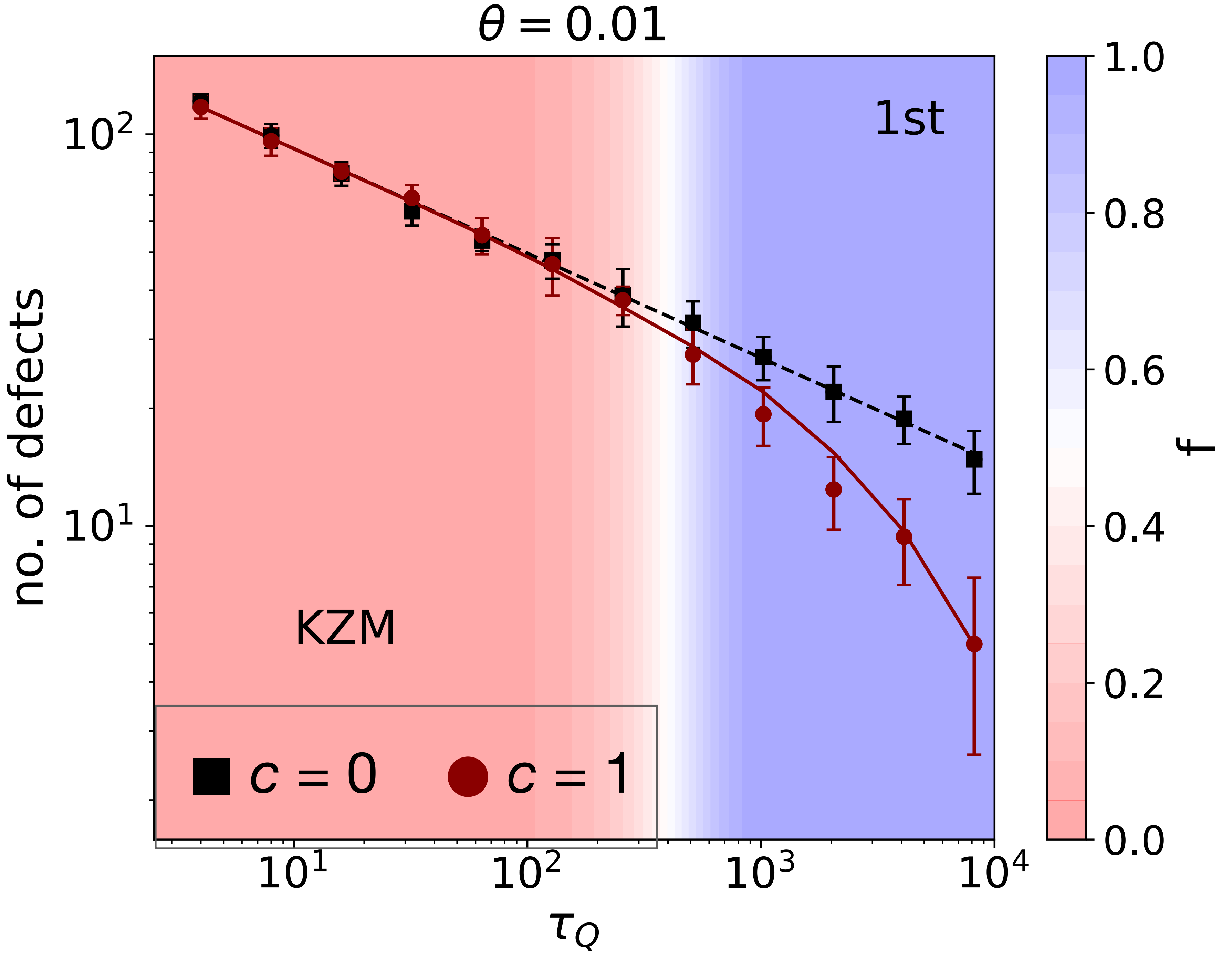
V_B has weaker deviation from KZM
(Nucleation barrier is higher,
nucleus growth velocity is smaller, $v \sim V_{old} - V_{new}$

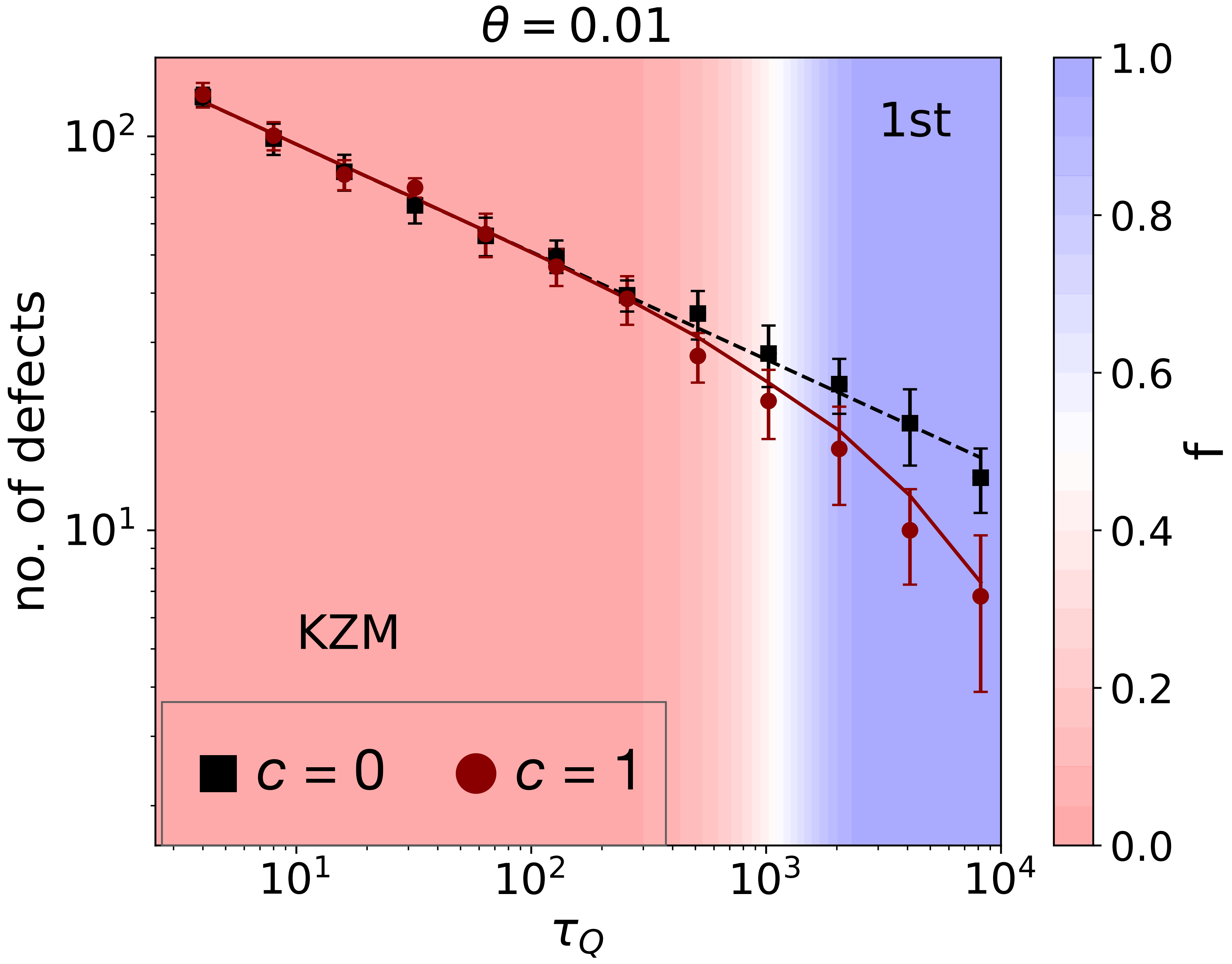
$$f = 1 - \exp(-\Omega)$$

$$\Omega = \int_{t_1}^{t_2} \Gamma(\epsilon(t))\mathcal{V}(t, t_2)dt \qquad \mathcal{V}(t, t_2) = \sigma \int_t^{t_2} v(\tau)d\tau$$

$$V_A(\phi) = (\phi^4 - 2\epsilon\phi^2)/8 - c|\phi|^3/3$$

$$V_B(\phi) = \phi^6/12 - \epsilon\phi^2/4 - c\phi^4/4$$ (Avadh Saxena)





Conclusion

-Weakly first order phase transition
can appear in superconducting phase transition/liquid crystals
(Fredericks phase transition)

-Weakly first order phase transition
↔first-order phase transition (nucleation) + second order phase transition (KZM)

# of defects

$$n = f n_{nuc} + (1 - f) n_{KZM}$$

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