


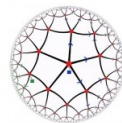
A postquantum theory of classical gravity?



WITH
Andrzej Grudka, Isaac Layton, Muhammad Sajjad,
Barbara Soda, Andrea Russo, Zach Weller-Davies

Phys. Rev. X 13, 041040 (2023)
Nature Comms 14, 7910 (2023)
arXiv:2302.07283
arXiv:2402.19459
arXiv:2402.17844

Jonathan Oppenheim
@postquantum 
QMCOSMOS, LANL
June 6, 2024



It from Qubit
Simons Collaboration on
Quantum Fields, Gravity and Information



**Engineering and
Physical Sciences
Research Council**



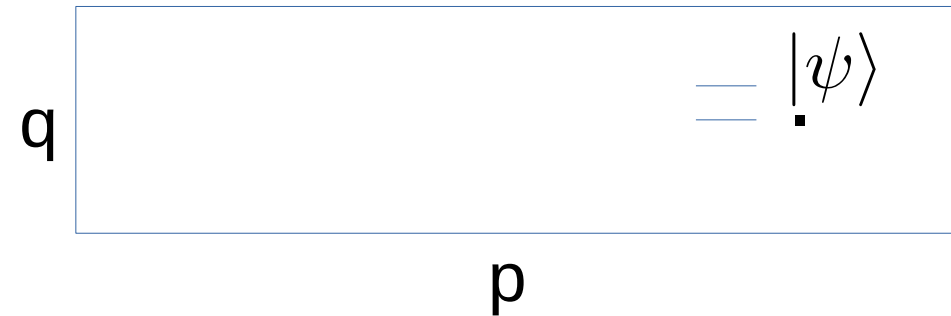
Frameworks

Quantum Mechanics

$$\frac{\partial \hat{\sigma}}{\partial t} = -i[\hat{H}, \hat{\sigma}]$$

Classical Mechanics

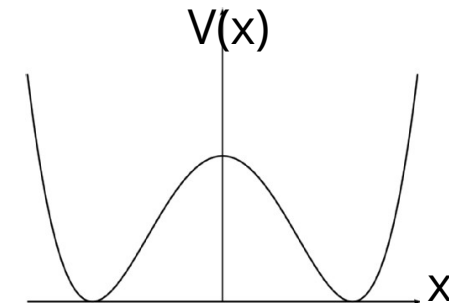
$$\frac{\partial \rho(q,p)}{\partial t} = \{H(q,p), \rho(q,p)\}$$



Frameworks

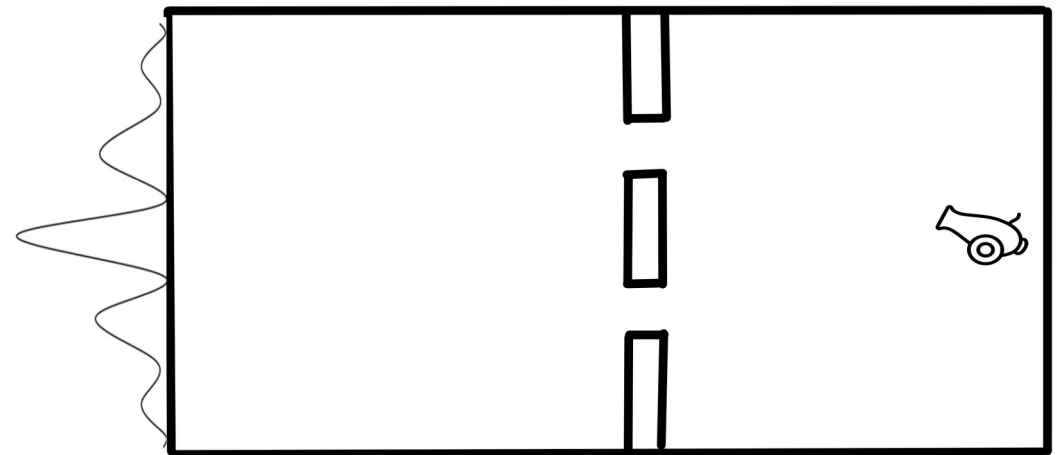
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$$\frac{\partial \hat{\sigma}}{\partial t} = -i[\hat{H}, \hat{\sigma}]$$



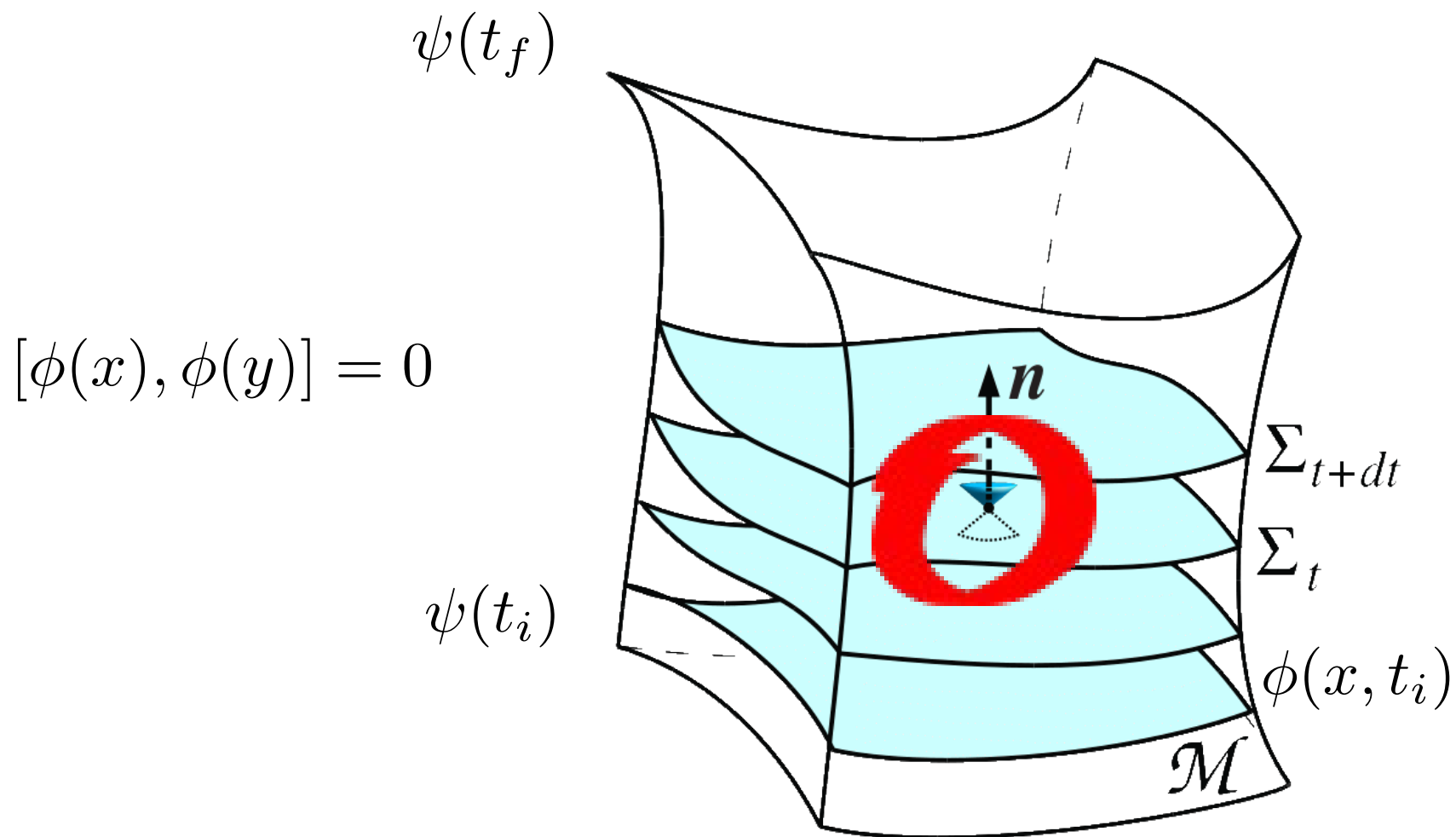
Classical Mechanics

$$\frac{\partial \rho(q,p)}{\partial t} = \{H(q,p), \rho(q,p)\}$$



Back-reaction

Motivation: fundamental or effective?



Motivation: gravity is unlike (other?) gauge theories

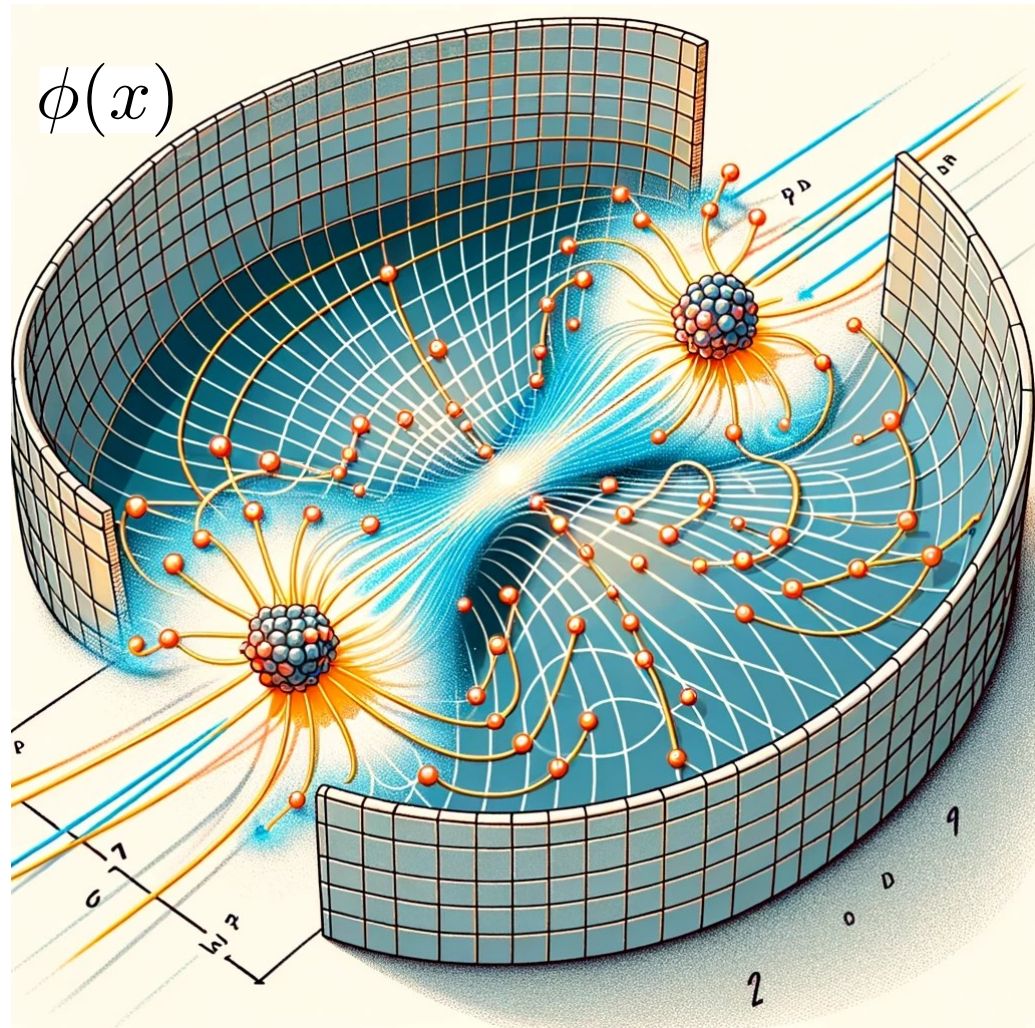
Not an internal gauge
degree of freedom at
 x, t

Universal geometry on
which fields live

Wheeler-deWitt Eqn

$$\mathcal{H}(x)|\psi\rangle = 0$$

Non-renormalisable
(quantum theory)



Classical-quantum gravity

If true

**Renormalisable
without Ghosts**

Black hole information
problem?

Small Λ ?
dark matter?

EXPERIMENT

Born rule & No need for
Measurement postulate

Decoherence vs
Diffusion trade-off

EXPERIMENT

Classical-quantum dynamics

Debate

No

Feynman (1957)
DeWitt (1962)
Unruh (1984)
Aharonov (~1986)
Eppley & Hannah (1977)
Unruh (1984)
Caro & Salcedo (1999)
Terno (2004)
Carlip (2008)
Marletto Vedral (2017)
Galley, Giacomini, Selby (2022)

Maybe

Sherry & Sudarshan (1978)
Boucher & Traschen (1988)
Kapral (1999)
Peres & Terno (2001)
Hall & Reginatto (2005)
Mattingly (2006)
Albers, Kiefer & Reginatto (2008)
Kent (2018)

Classical-quantum dynamics

History

Semi-classical Einstein
(pathological when
fluctuations are large)

Page & Geilker (1981);
Gisin (1989)

Simple examples

Blanchard & Jadczyk (1994);
Diosi (1995);
Poulin (2017);

Kafri, Taylor, Milburn (2014);
Diosi, Tilloy (2016)

Quantum chemistry
(negative probabilities)

Kapral review (2006);
Koopman-von Neumann (1931-32)

Experiments!

Kafri & Taylor (2013);
Bose et. al. (2017);
Marletto et. al. (2017)
Lami, Pedarnals, Plenio (2022)
Carney (2108.06320)

A post-quantum theory of classical gravity?

What is the most general form of CQ dynamics?

Decoherence vs Diffusion: testing quantum gravity

Renormalisable without Ghosts!

Anomalous contribution to the metric (dark matter, dark energy?)

A post-quantum theory of classical gravity?

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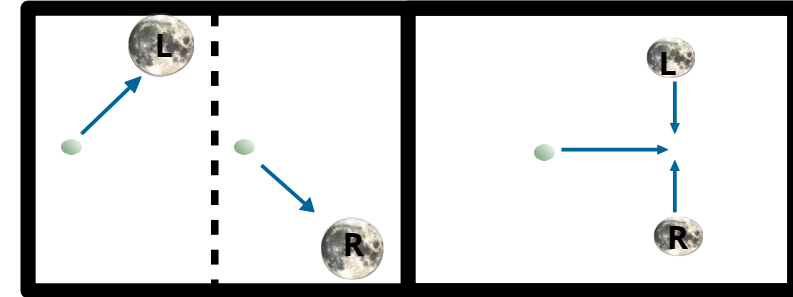
Renormalisable without Ghosts!

Anomalous contribution to the metric (dark matter, dark energy?)

Semi-classical equations are pathological

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \langle \hat{T}_{\mu\nu} \rangle$$

$$\rho^{(gm)} = \frac{1}{2} \rho_L^{(g)} \rho_L^{(m)} + \frac{1}{2} \rho_R^{(g)} \rho_R^{(m)}$$



This or that

Not this!

Consistent Dynamics: linear and preserve state-space (completely positive and norm preserving)

Requires decoherence and diffusion

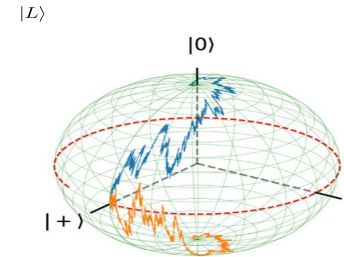
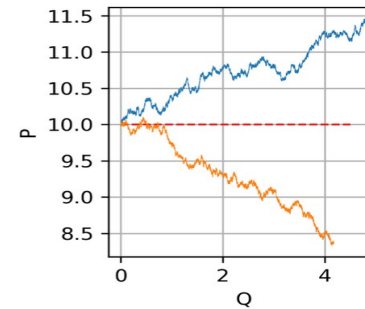
CQ Dynamics

Path Integral

$$\rho(q, p, \phi^\pm, t_f) = \int \mathcal{D}q \mathcal{D}p \mathcal{D}\phi^\pm e^{iS_C[q, p] + iS[\phi^+] - iS[\phi^-] + iS_{FV}[\phi^\pm] + iS_{CQ}[q, p, \phi^\pm]} \delta\left(\dot{q} - \frac{p}{m}\right) \rho(q, p, \phi^\pm, t_i)$$

JO, Zach Weller-Davies

Trajectories



JO, I. Layton, Z. Weller-Davies

Master Eqn

CPTP MAP

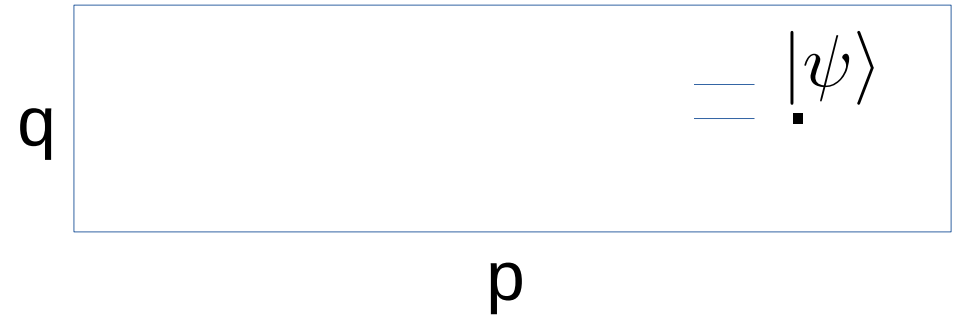
$$\begin{aligned} \frac{\partial \hat{\rho}}{\partial t} \approx & \{H^{(grav)}, \hat{\rho}\} - i[\hat{H}^{(m)}, \hat{\rho}] + \frac{1}{2}\{\hat{H}^{(m)}, \hat{\rho}\} - \frac{1}{2}\{\hat{\rho}, \hat{H}^{(m)}\} \\ & + \int dx dx' \frac{\delta^2}{\delta\pi_\Phi(x)\delta\pi_\Phi(x')} (D_2(x, x')\hat{\rho}) + \frac{1}{2} \int dx dx' D_0(x, x') ([\hat{m}(x), [\hat{\rho}, \hat{m}(x')]]) \end{aligned}$$

JO, Sparaciari, Soda, Weller-Davies

Frameworks

Quantum Mechanics

$$\frac{\partial \hat{\sigma}}{\partial t} = -i[\hat{H}, \hat{\sigma}]$$



Classical Mechanics

$$\frac{\partial \rho(q,p)}{\partial t} = \{H(q,p), \rho(q,p)\}$$

$$\hat{\rho}(z) = \rho(z) \begin{pmatrix} p(0|z) & \alpha(z) \\ \alpha^*(z) & p(1|z) \end{pmatrix}$$

Classical, quantum, & CQ States

Q

HILBERT SPACE

 $\hat{\sigma}$

$$\text{tr } \hat{\sigma} = 1$$

POSITIVE MATRIX

C

PHASE SPACE

 $\rho(q, p)$

$$\int dq dp \rho(q, p) = 1$$

POSITIVE DISTRIBUTION

CQ

$$\hat{\rho}(z; t) = \rho(z; t) \hat{\sigma}(z; t)$$

$$z := (q, p)$$

$$\int dz \text{tr } \hat{\rho}(z) = 1$$

 $\hat{\rho}(z)$

POSITIVE MATRIX AT EACH Z

Classical, quantum, & CQ States

Q

HILBERT SPACE

$$\text{tr } \hat{\sigma} = 1 \quad \hat{\sigma} = \begin{pmatrix} p(0) & \alpha \\ \alpha^* & p(1) \end{pmatrix}$$

POSITIVE MATRIX

C

PHASE SPACE

$$\int dqdp \rho(q, p) = 1$$

POSITIVE DISTRIBUTION

CQ

$$\hat{\sigma}_{cq} = \int dz \rho(z; t) |z\rangle\langle z| \otimes \sigma(z; t)$$

$$\hat{\rho}(z) = \rho(z) \begin{pmatrix} p(0|z) & \alpha(z) \\ \alpha^*(z) & p(1|z) \end{pmatrix}$$

$$\int dz \text{tr } \hat{\rho}(z) = 1$$

POSITIVE MATRIX AT EACH Z

Dynamics must be linear and preserve state-space

$$\mathbb{1} \otimes \mathcal{L}$$

Must be positive

Norm preserving

positive matrix at each z

$$\int dz \operatorname{tr} \hat{\rho}(z) = 1$$

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle|\psi_L\rangle - |1\rangle|\psi_R\rangle) \quad \sigma^{(m)} = \frac{1}{2} |\psi_L\rangle\langle\psi_L| + \frac{1}{2} |\psi_R\rangle\langle\psi_R|$$

$$\frac{1}{2} \mathcal{L}(|\psi_L\rangle\langle\psi_L|) + \frac{1}{2} \mathcal{L}(|\psi_R\rangle\langle\psi_R|) = \mathcal{L}\left(\frac{1}{2} |\psi_L\rangle\langle\psi_L| + \frac{1}{2} |\psi_R\rangle\langle\psi_R|\right)$$

Path Integrals

Quantum Mechanics

$$\langle \phi^-, t_f | \phi_i, t_i \rangle \langle \phi_i, t_i | \phi^+, t_f \rangle = \int_{\phi^+ = \phi^- = \phi_i}^{\phi_f^+, \phi_f^-} \mathcal{D}\phi^+ \mathcal{D}\phi^- e^{iS[\phi^+] - iS[\phi^-]}$$

Path Integrals

Quantum Mechanics

$$\hat{\sigma}(\phi_f^+, \phi_f^-, t_f) = \int_{\phi^+ = \phi^- = \phi_i}^{\phi_f^+, \phi_f^-} \mathcal{D}\phi^+ \mathcal{D}\phi^- e^{iS[\phi^+] - iS[\phi^-]}$$

Path Integrals

Quantum Mechanics (open systems)

$$\hat{\sigma}(\phi_f^+, \phi_f^-, t_f) = \int \mathcal{D}\phi^+ \mathcal{D}\phi^- e^{iS[\phi^+] - iS[\phi^-] + iS_{FV}[\phi^+, \phi^-]}$$

$$iS_{FV} = D_0 \int_{t_i}^{t_f} d^4x \left(\phi^+ \phi^- - \frac{1}{2} (\phi^- \phi^- + \phi^+ \phi^+) \right)$$

Feynman-Vernon

Path Integrals

Quantum Mechanics (open systems)

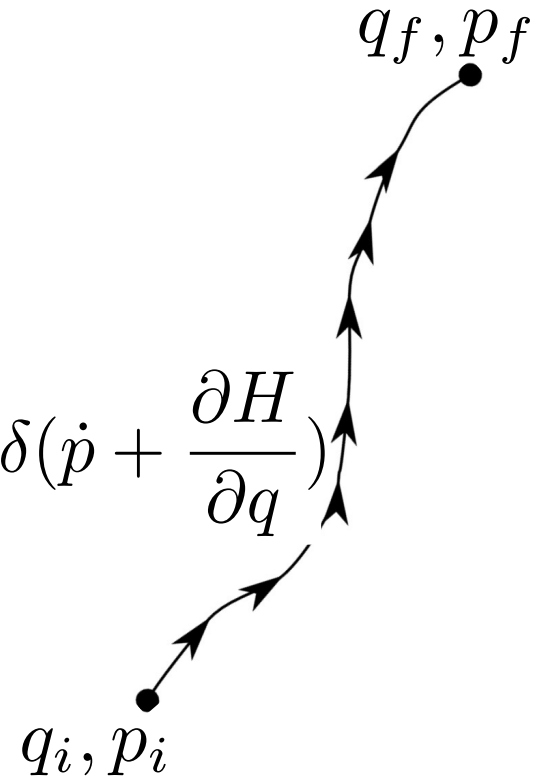
$$\hat{\sigma}(\phi_f^+, \phi_f^-, t_f) = \int \mathcal{D}\phi^+ \mathcal{D}\phi^- e^{iS[\phi^+] - iS[\phi^-] + iS_{FV}[\phi^+, \phi^-]}$$

$$iS_{FV} = -\frac{1}{2}D_0 \int_{t_i}^{t_f} d^4x (\phi^+ - \phi^-)^2$$

Path Integrals

Classical Mechanics

$$\rho(q_f, p_f, t_f | q_i, p_i, t_i) = \int_{q_i, p_i}^{q_f, p_f} \mathcal{D}q \mathcal{D}p \Pi_t \delta(\dot{q} - \frac{\partial H}{\partial p}) \delta(\dot{p} + \frac{\partial H}{\partial q})$$



Path Integral for Stochastic GR

“Equation of Motion Squared”

$$\mathcal{I} = -\frac{1}{2D_2} \int dt \left(\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} \right)^2$$

$$\mathcal{I} = -\frac{1}{2D_2} \int dt \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right)$$

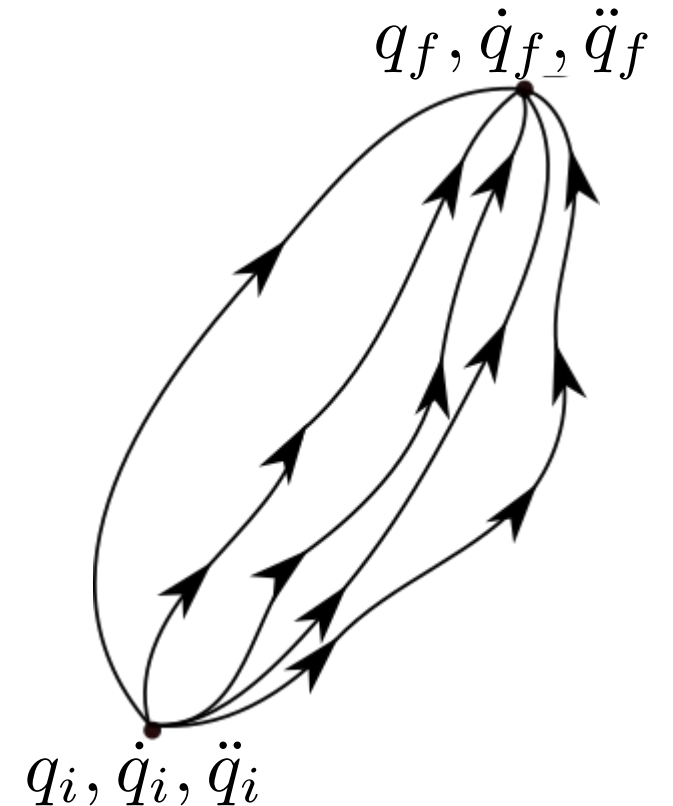
Brownian motion

Classical Mechanics (stochastic)

$$\ddot{q} = \frac{F}{m} + j(t)$$

$$\langle j(t) \rangle = 0, \quad \langle j(t)j(s) \rangle = D_2 \delta(s, t)$$

Langevin Eqn (or Fokker-Planck, or Ito)



Q, C & CQ Path integral

Q

$$iS_Q[\phi^+, \phi^-] := iS[\phi^+] - iS[\phi^-] + iS_{FV}[\phi^+, \phi^-]$$

$$iS_{FV} = -\frac{1}{2}D_0 \int_{t_i}^{t_f} d^4x (\phi^+ - \phi^-)^2$$

C

$$iS_C[q, p] = -\frac{1}{2D_2} \int dt (\dot{p} + \frac{\partial H}{\partial q})^2$$

Onsager, Machlup (1953);
Freidlin, Wentzell (1998)

CQ

$$iS[\phi^+, \phi^-, q, p] = S_Q[\phi^+, \phi^-] - \frac{1}{2D_2} \int dt (\dot{p} + \frac{1}{2} \frac{\partial \hat{H}^+}{\partial q} + \frac{1}{2} \frac{\partial \hat{H}^-}{\partial q})^2$$

$$iS_{FV} = -\frac{1}{2} \int dt D_0 \left(\frac{\partial \hat{H}^+}{\partial q} - \frac{\partial \hat{H}^-}{\partial q} \right)^2$$

$$4D_2 \succeq D_0^{-1}$$

Q, C & CQ Dynamics (jumps)

Q

$$\frac{\partial \hat{\sigma}}{\partial t} = -i[\hat{H}, \hat{\sigma}] + \sum_{\alpha\beta} W^{\alpha\beta} \hat{L}_\alpha \hat{\sigma} \hat{L}_\beta^\dagger - \frac{1}{2} W^{\alpha\beta} \{ \hat{L}_\beta^\dagger \hat{L}_\alpha, \hat{\sigma} \}_+$$

$$W^{\alpha\beta} \succeq 0$$

C

$$\frac{\partial \rho(z)}{\partial t} = \{H(z), \rho(z)\} + \int dz' (W(z|z')\rho(z') - W(z)\rho(z))$$

$$W(z|z') \geq 0 \quad \forall z, z'$$

CQ

$$\frac{\partial \hat{\rho}(z;t)}{\partial t} = -i[\hat{H}(z), \hat{\rho}(z;t)] + \sum_{\alpha\beta} \int dz' W^{\alpha\beta}(z|z') \hat{L}_\alpha \hat{\rho}(z';t) \hat{L}_\beta^\dagger - \frac{1}{2} W^{\alpha\beta}(z) \{ \hat{L}_\beta^\dagger \hat{L}_\alpha, \hat{\rho}(z;t) \}$$

$$W^{\alpha\beta} \succeq 0 \quad \forall z, z'$$

Blanchard, Jazdyk (1994);
Poulin (2017);
JO (2018)

Q, C & CQ Dynamics (continuous)

Q

$$\frac{\partial \hat{\sigma}}{\partial t} = -i[\hat{H}, \hat{\sigma}] + \sum_{\alpha\beta} W^{\alpha\beta} \hat{L}_\alpha \hat{\sigma} \hat{L}_\beta^\dagger - \frac{1}{2} W^{\alpha\beta} \{\hat{L}_\beta^\dagger \hat{L}_\alpha, \hat{\sigma}\}_+$$

$$W^{\alpha\beta} \succeq 0$$

C

$$\frac{\partial \rho(z)}{\partial t} = - \sum_i \frac{\partial}{\partial z_i} (D_{1,i} \rho(z)) + \sum_{ij} \frac{\partial^2}{\partial z_i \partial z_j} (D_{2,ij} \rho(z))$$

$$D_2(z) \succeq 0$$

Fokker-Planck Equation

CQ

$$\begin{aligned} \frac{\partial \hat{\rho}(z)}{\partial t} = & -i[H(z), \hat{\rho}(z)] + \mathcal{L}_{FP} \hat{\rho}(z) + \frac{\partial}{\partial z_i} (D_{1,i}^{0\alpha} \hat{\rho}(z) L_\alpha^\dagger) + \frac{\partial}{\partial z_i} (D_{1,i}^{\alpha 0} L_\alpha \hat{\rho}(z)) \\ & + D_0^{\alpha\beta}(z) L_\alpha \hat{\rho}(z) L_\beta^\dagger - \frac{1}{2} D_0^{\alpha\beta} \{L_\beta^\dagger L_\alpha, \hat{\rho}(z)\}_+ \end{aligned}$$

$$D_2(z) - D_1^\dagger(z) D_0^{-1}(z) D_1(z) \succeq 0$$

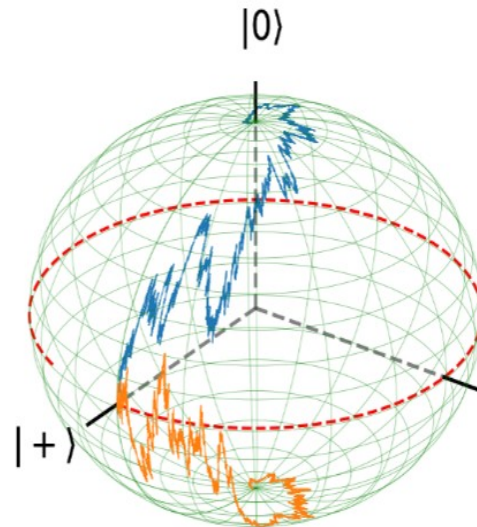
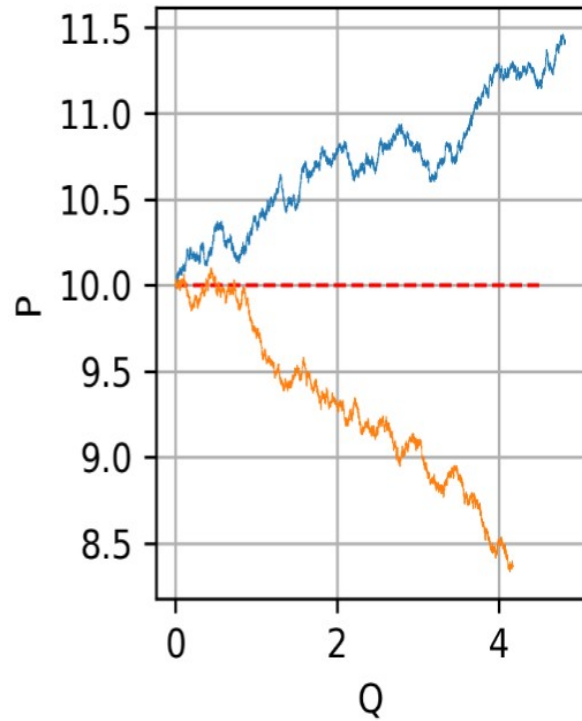
JO (2018);

JO, Soda, Sparaciari, Weller-Davies (2022)

Example of continuous master-equation

Stern-Gerlach

$$\hat{H} = \frac{p^2}{2m} + q\hat{\sigma}$$

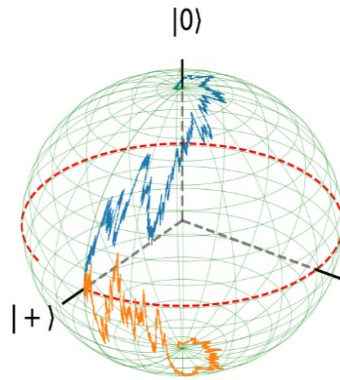
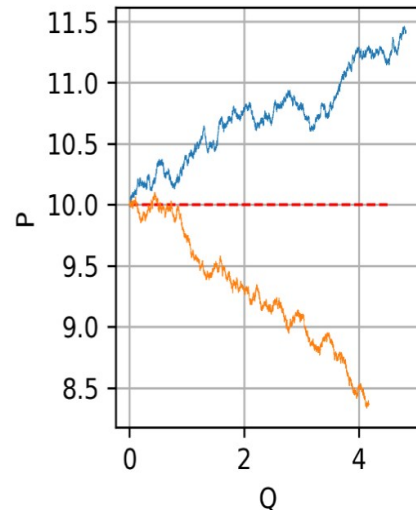


$$D_2 D_0 \succeq D_1^2$$

Example of continuous master-equation

Stern-Gerlach $\hat{H} = \frac{p^2}{2m} + D_1 q \hat{\sigma}$

$$\frac{\partial \hat{\rho}}{\partial t} = -i[\hat{H}, \hat{\rho}] + \frac{1}{2}\{\hat{H}, \hat{\rho}\} - \frac{1}{2}\{\hat{\rho}, \hat{H}\} + \frac{D_2}{2}\{q, \{q, \hat{\rho}\}\} + \frac{D_0}{2}[\hat{\sigma}, [\hat{\rho}, \hat{\sigma}]]$$



$$D_2 D_0 \succeq D_1^2$$

Diosi (1995)

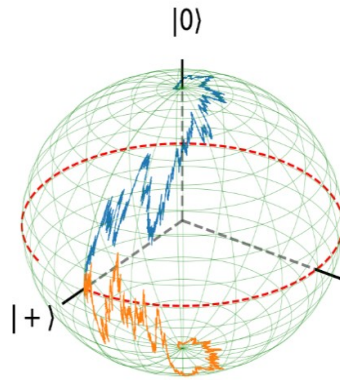
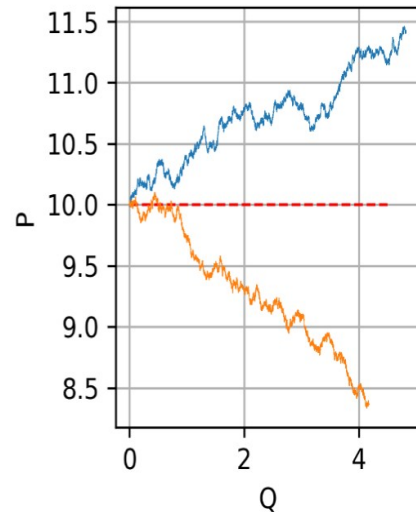
Isaac Layton, JO, Zach Weller-Davies (2022)

Example of continuous master-equation

Stern-Gerlach

$$\hat{H} = \frac{p^2}{2m} + D_1 q \hat{\sigma}$$

$$\frac{\partial \hat{\rho}}{\partial t} = -i[\hat{H}, \hat{\rho}] + \frac{D_1}{2} \left(\hat{\sigma} \frac{\partial \hat{\rho}}{\partial p} + \frac{\partial \hat{\rho}}{\partial p} \hat{\sigma} \right) + \frac{D_2}{2} \frac{\partial^2 \hat{\rho}}{\partial p^2} + \frac{D_0}{2} [\hat{\sigma}, [\hat{\rho}, \hat{\sigma}]]$$



$$D_2 D_0 = D_1^2$$

Diosi (1995)
Isaac Layton, JO, Zach Weller-Davies

A post-quantum theory of classical gravity?

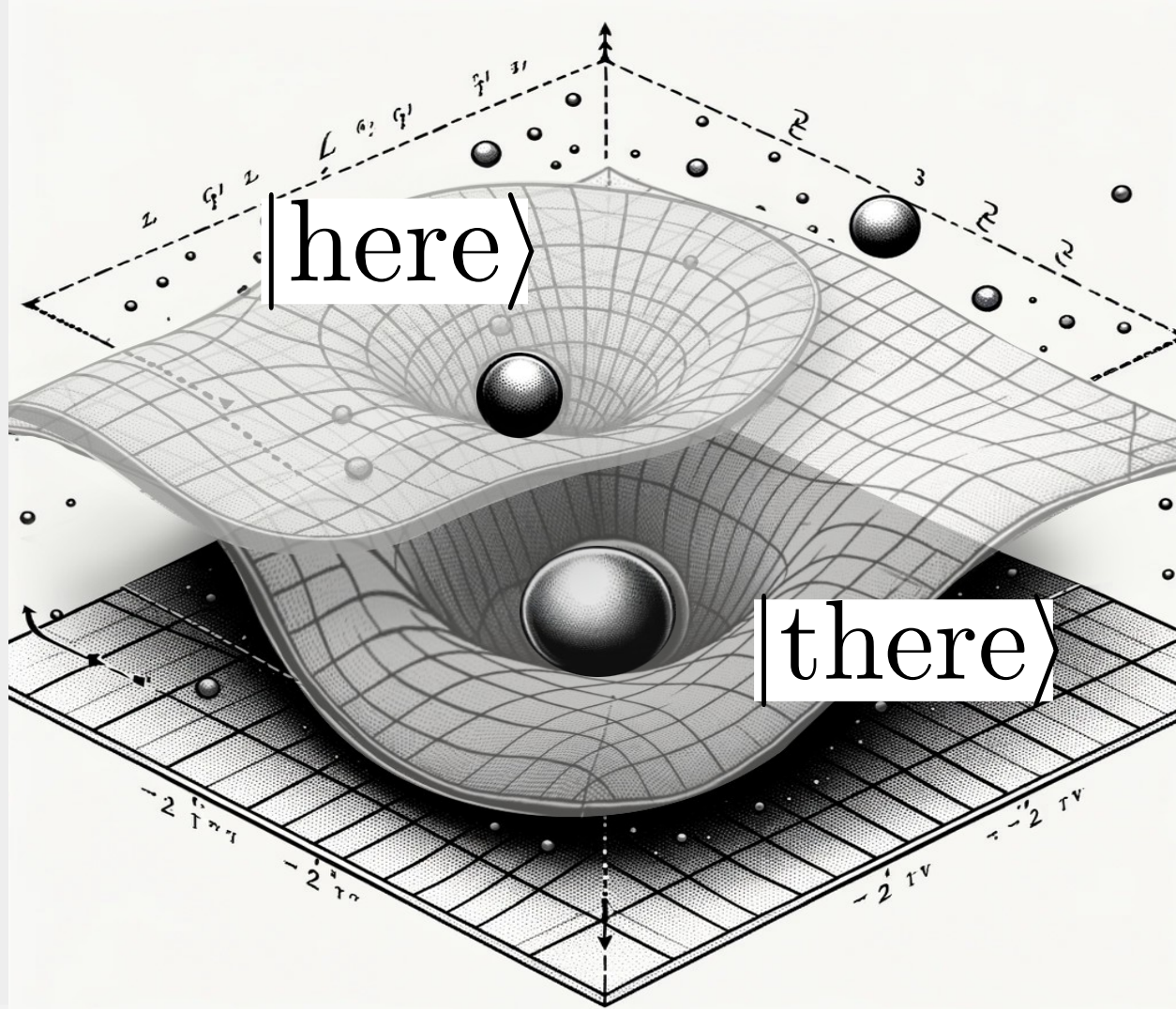
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Decoherence vs Diffusion: testing quantum gravity

Renormalisable without Ghosts!

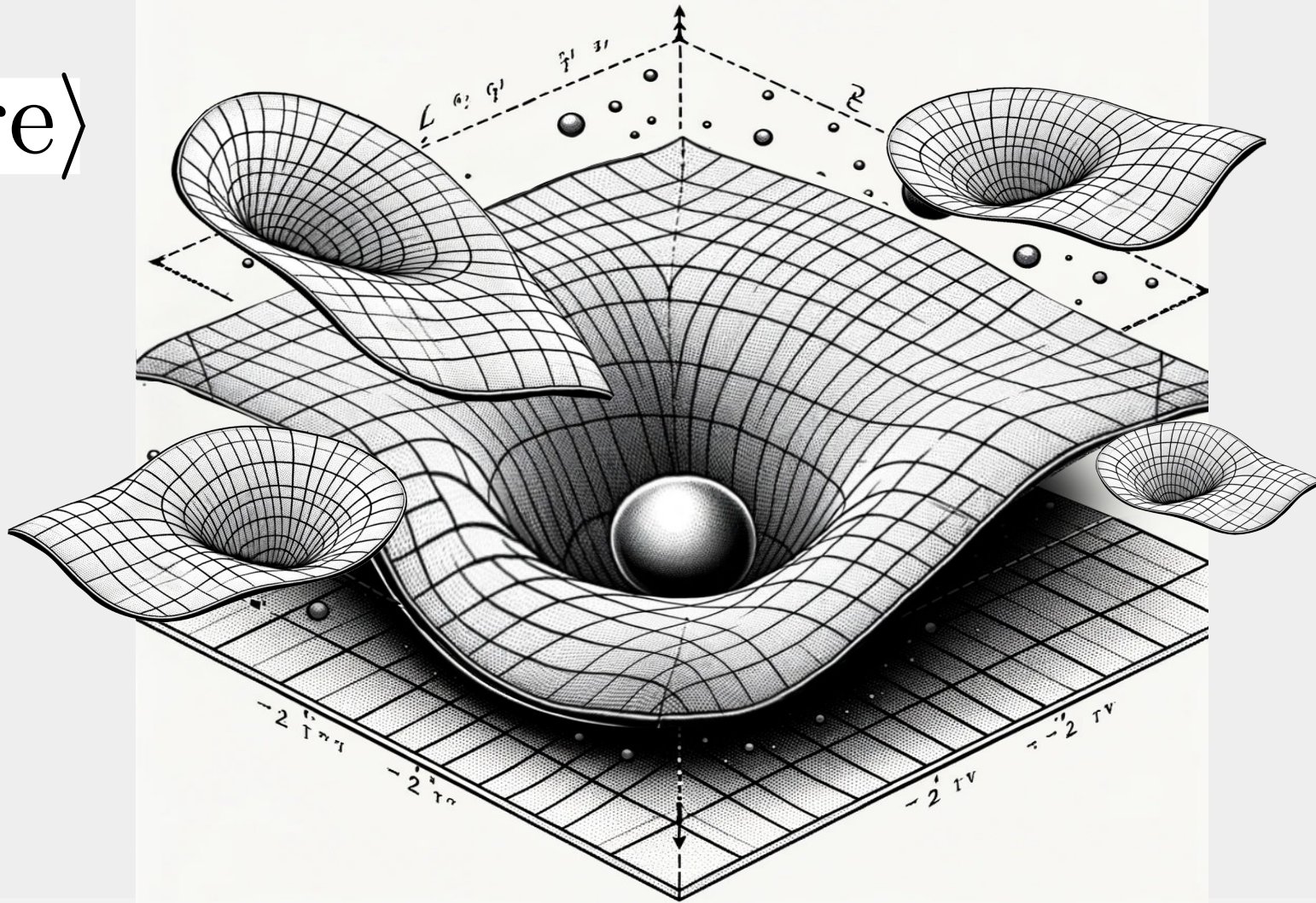
Anomalous contribution to the metric (dark matter, dark energy?)

Decoherence vs diffusion trade-off



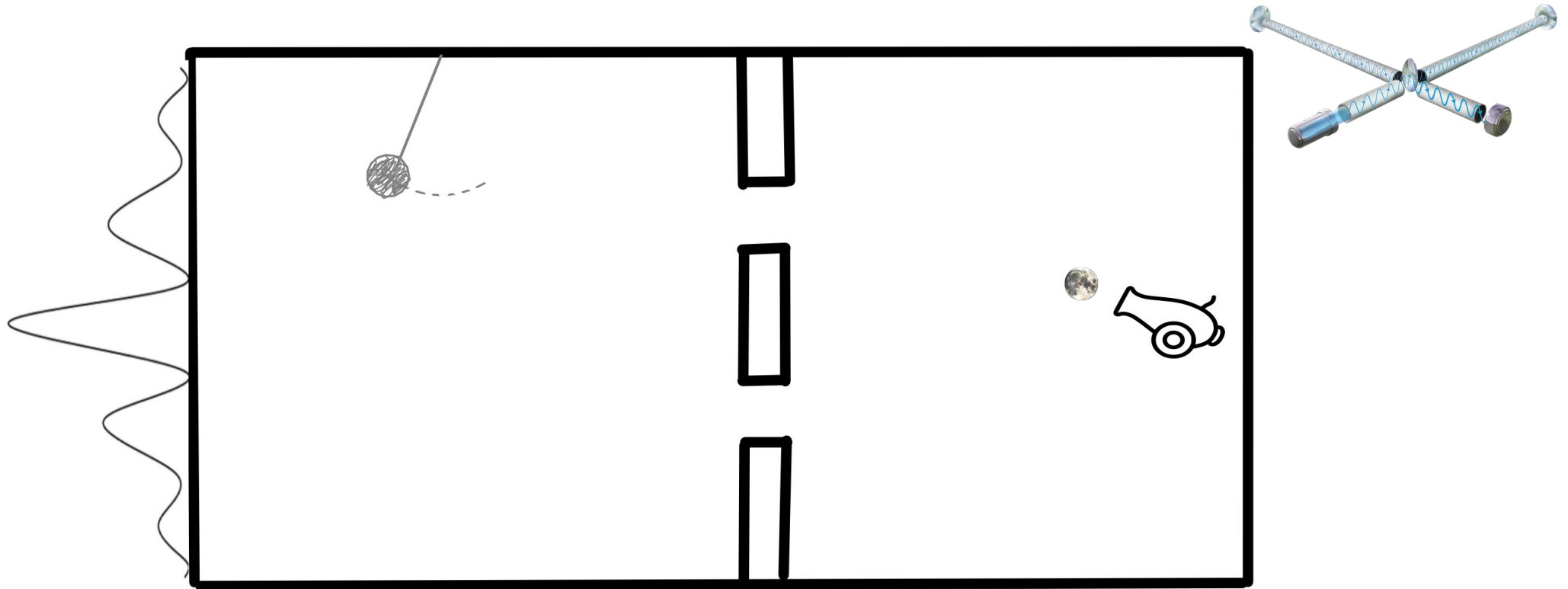
Decoherence vs diffusion trade-off

$| \text{here} \rangle$



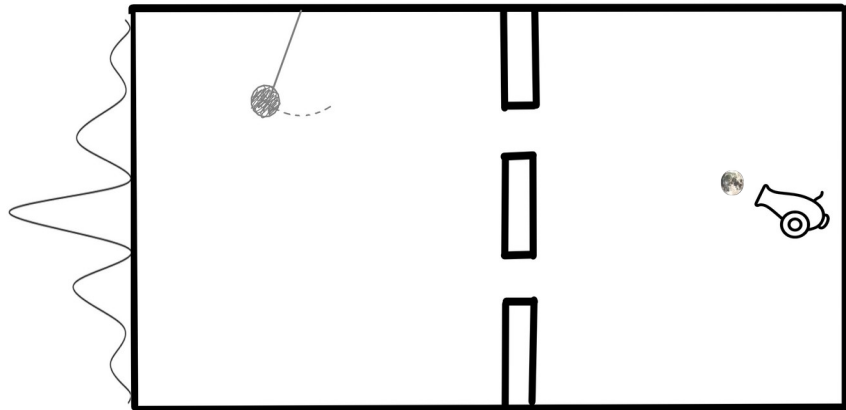
$| \text{there} \rangle$

Decoherence vs diffusion



$$|\psi\rangle = \frac{1}{\sqrt{2}} (|E_L\rangle |L\rangle + |E_R\rangle |R\rangle)$$

Decoherence vs diffusion

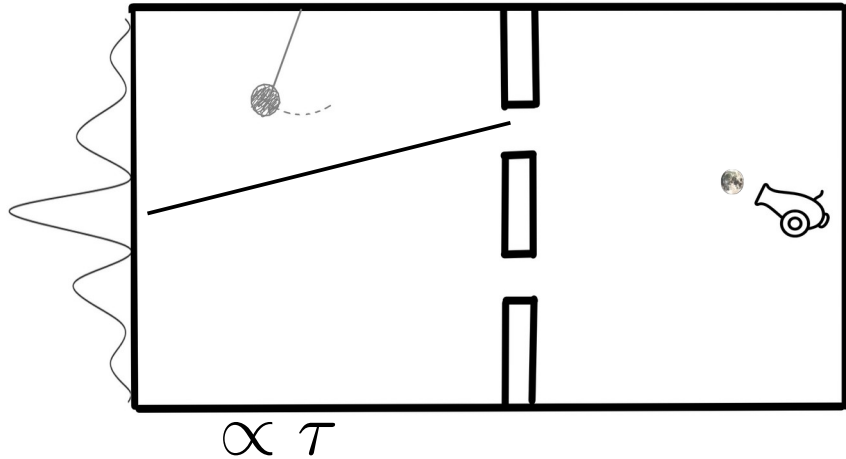


$$\hat{\sigma}(t) = \begin{pmatrix} \frac{1}{2} & \alpha^*(t) \\ \alpha(t) & \frac{1}{2} \end{pmatrix}$$

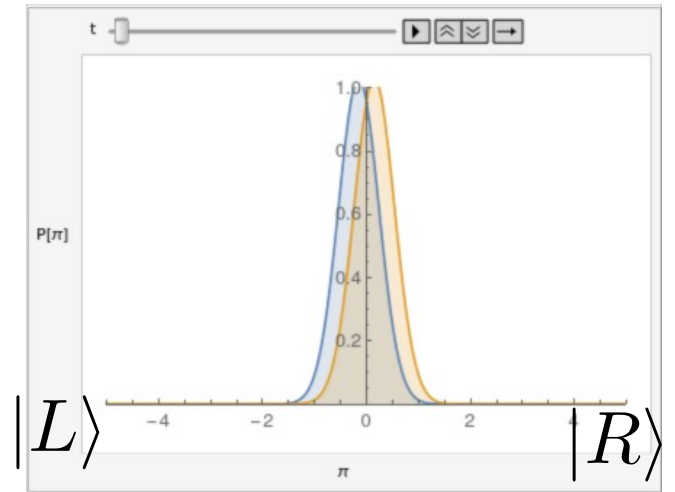
$$\alpha(t) = \langle E_L(t) | E_R(t) \rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|E_L\rangle |L\rangle + |E_R\rangle |R\rangle)$$

Decoherence vs diffusion



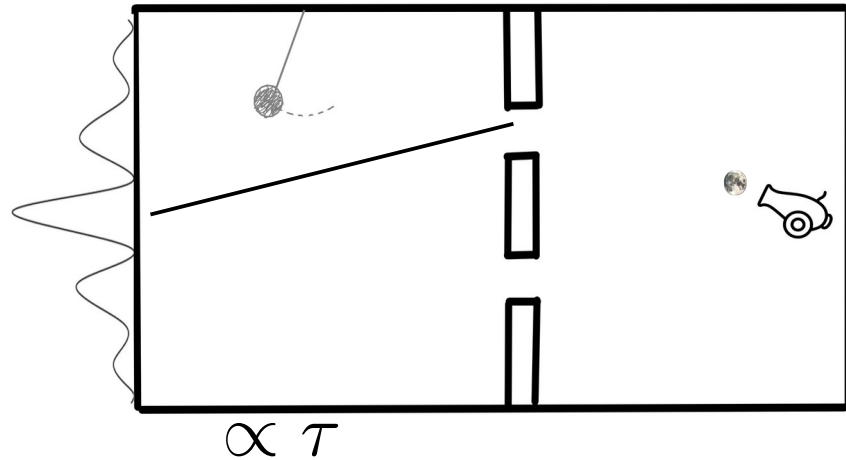
$$\tau \leq \frac{D_2}{\langle F \rangle^2}$$



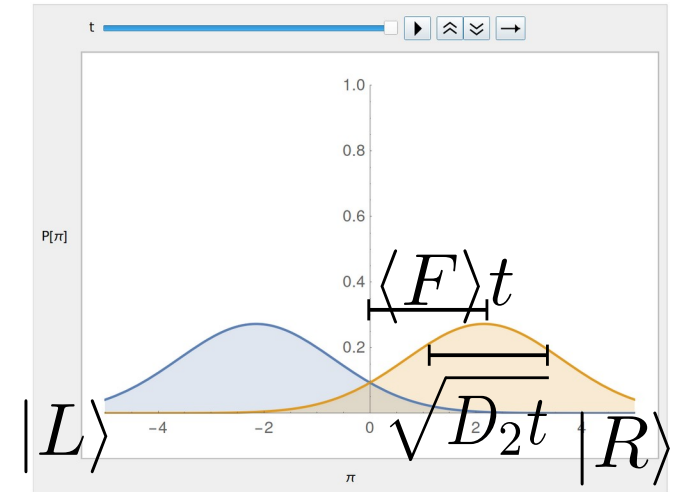
$$D_2(z) - D_1^\dagger(z) D_0^{-1}(z) D_1(z) \succeq 0$$

Holds for all classical-quantum dynamics

Decoherence vs diffusion



$$\tau \leq \frac{D_2}{\langle F \rangle^2}$$

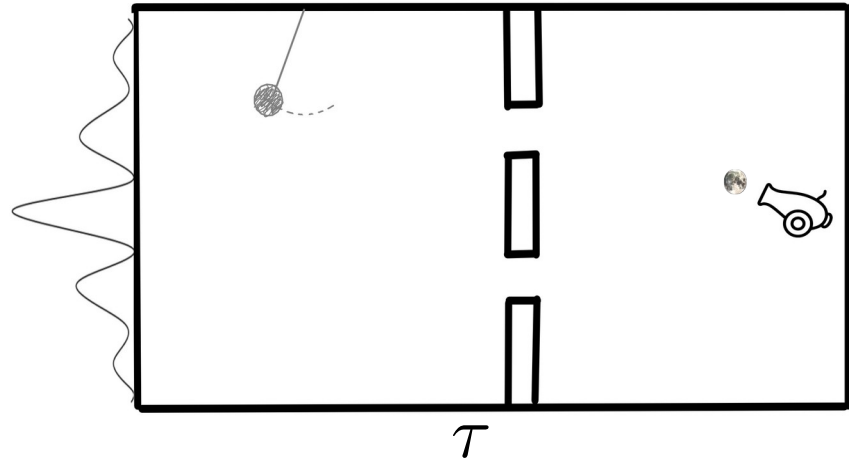


$$D_2(z) - D_1^\dagger(z) D_0^{-1}(z) D_1(z) \succeq 0$$

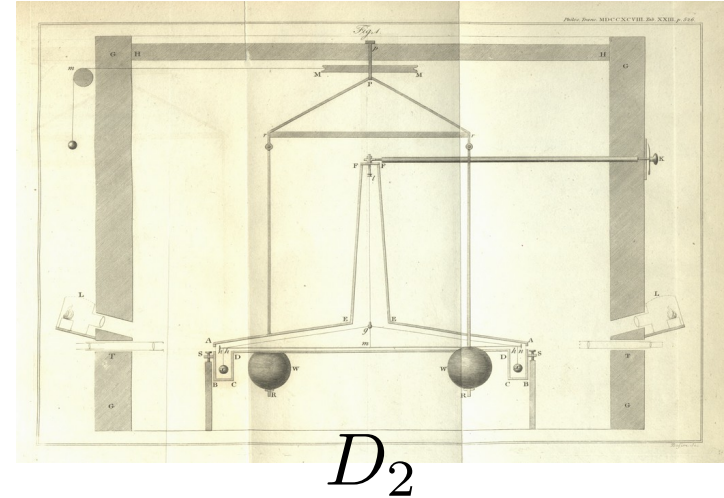
Holds for all classical-quantum dynamics

Decoherence vs diffusion

Double Slit Experiment



Cavendish Experiment



$$\langle F(x) \rangle = \langle \hat{m}(x) \rangle$$

$$D_2(z) - D_1^\dagger(z) D_0^{-1}(z) D_1(z) \succeq 0$$

$$\tau \leq \frac{D_2}{\langle F \rangle^2}$$

A post-quantum theory of classical gravity?

What is the most general form of CQ dynamics?

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Path Integrals

Quantum Mechanics

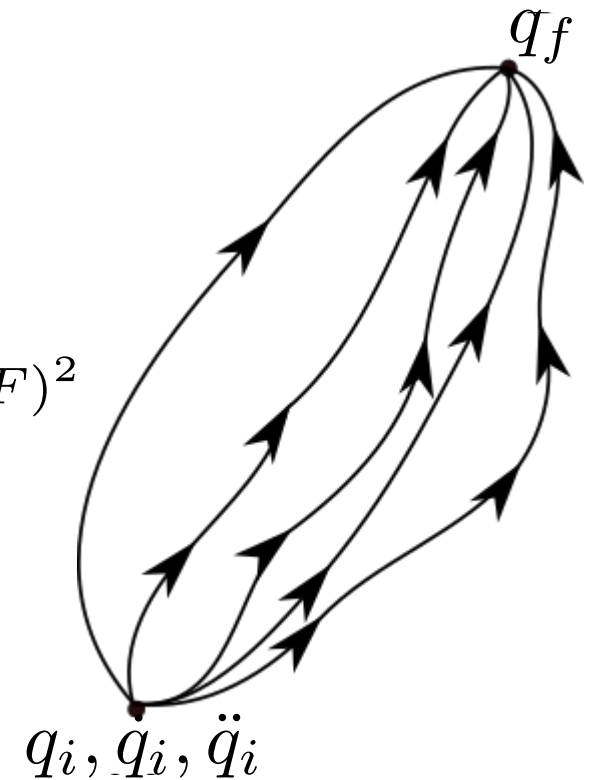
$$\langle q_f, t_f | q_i, t_i \rangle = \int_{q_i}^{q_f} \mathcal{D}q e^{i \int dt \left(\frac{1}{2} m \dot{q}^2 - V(q) \right)}$$



Path Integral for Brownian motion

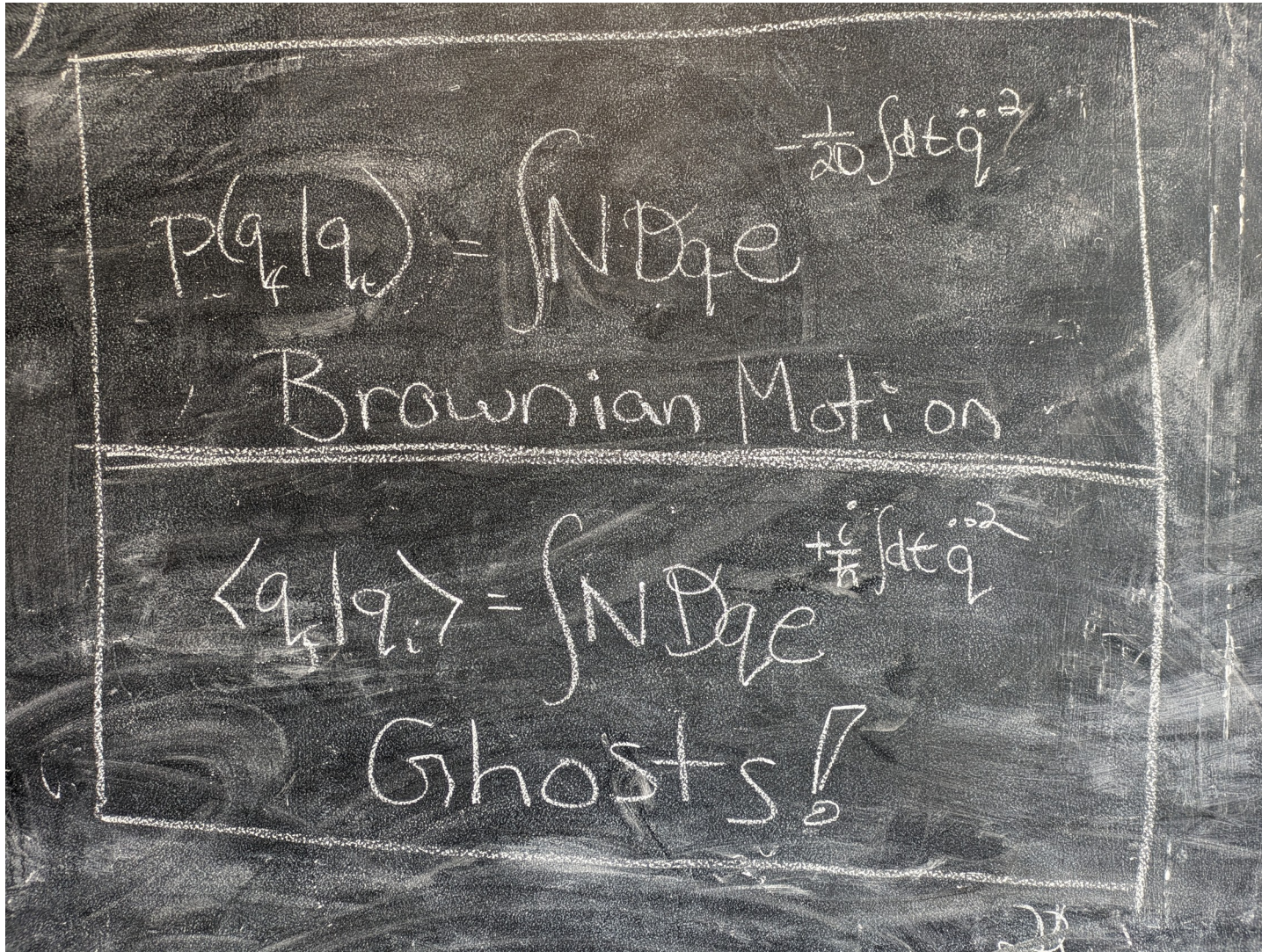
Classical Mechanics (stochastic)

$$\rho(q_f, t_f | q_i, \dot{q}_i, \ddot{q}_i, t_i) = \mathcal{N} \int_{q_i, \dot{q}_i, \ddot{q}_i}^{q_f} \mathcal{D}q e^{-\frac{1}{2D_2} \int dt (m\ddot{q} - F)^2}$$



Onsager-Machlup

Main Message!



No Ostrogradsky Instability

$$\mathcal{I}_{OM} = -\frac{1}{2D_2} \int dt (m\ddot{q} - F)^2$$

$$\mathcal{I}_{QM} = \frac{i}{\hbar} \int dt (m\ddot{q} - F)^2$$

$$H = \frac{p^2}{2m} + V(q) + j(t)$$

$$H = \frac{D_2 P_1^2}{2m^2} + P_1 X_2 + \frac{F P_2}{m}$$

Path Integral for Stochastic GR

Stochastic GR vs Quadratic Gravity

$$\mathcal{I} = -\frac{1}{2D_2} \int dt \left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \right) \left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R \right)$$

$$\mathcal{I} = -\frac{1}{2D_2} \int dt \left(R_{\mu\nu}R^{\mu\nu} + |\beta|R^2 \right)$$

Grudka, JO, Sajjad, Russo,
2402.17844

Path Integral for Stochastic GR

Stochastic GR vs Quadratic Gravity

$$\mathcal{I}^{(gr)} = - \int d^4x \left(\frac{1}{2D_2} R^2 + \frac{1}{2D_w} C^2 + \frac{1}{\alpha} \mathcal{G} \right)$$

$$\mathcal{I}^{(m)} = -\frac{G_N}{16\pi D_2} \int dx \sqrt{-g} \left[-2\bar{T}^{\mu\nu} R_{\mu\nu} + (1 - 2\beta) \bar{T} R - 8\pi G_N \beta \bar{T}^2 + 8\pi G_N \bar{T}^{\mu\nu} \bar{T}_{\mu\nu} \right]$$

Grudka, JO, Sajjad, Russo,
2402.17844

A post-quantum theory of classical gravity?

What is the most general form of CQ dynamics?

Decoherence vs Diffusion: testing quantum gravity

Renormalisable without Ghosts!

Anomalous contribution to the metric (dark matter, dark energy?)

Anomalous contribution (Brownian motion)

The most probably path (MPP)

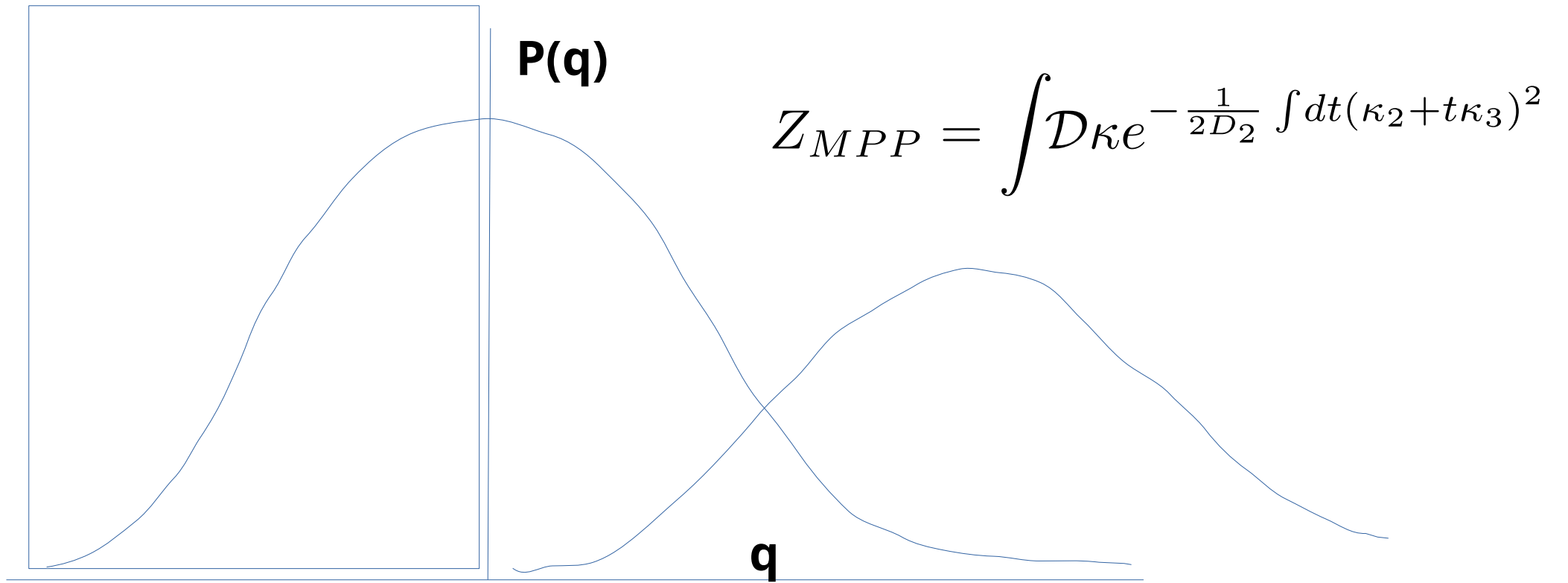
$$\mathcal{I} = -\frac{1}{2D_2} \int dt (m\ddot{q})^2$$

$$\frac{d^4 q}{dt^4} = 0$$

$$Z = \int \mathcal{D}q e^{-\frac{1}{2D_2} \int dt (m\ddot{q})^2 (\kappa_2 + t\kappa_3)^2}$$

$$q_{\text{MPP}} = \kappa_0 + \kappa_1 t + \frac{1}{2} \kappa_2 t^2 + \frac{1}{6} \kappa_3 t^3$$

Anomalous contribution (Brownian motion)



Anomalous contribution (Brownian motion)

The most probably path (MPP)

$$\mathcal{I} = -\frac{1}{2D_2} \int dt (m\ddot{q})^2$$

$$\frac{d^4 q}{dt^4} = 0$$

$$Z_{MPP} = \int \mathcal{D}\kappa e^{-\frac{1}{2D_2} \int dt (\kappa_2 + t\kappa_3)^2}$$

$$q_{MPP} = \kappa_0 + \kappa_1 t + \frac{1}{2} \kappa_2 t^2 + \frac{1}{6} \kappa_3 t^3$$

Anomalous contribution (Newtonian gravity)

The most probably path (MPP)

$$\mathcal{I} = -\frac{1}{2D_2} \int d^3x (\nabla^2 \Phi - 4\pi Gm)^2$$

$$\nabla^4 \Phi = 0$$

$$\Phi_{\text{MPP}} = -\frac{\kappa m}{r} + \kappa_1 8\pi r + \kappa_2 r^2$$

Anomalous contribution (Schwarschild-deSitter)

$$\mathcal{I}_\gamma = -\frac{6\pi VT}{D_2} \left(\frac{5 - 18\beta}{r_{\max}^2} \gamma_1^2 + 6(1 - 4\beta) \gamma_2^2 + \frac{9(1 - 4\beta)}{r_{\max}} \gamma_1 \gamma_2 \right)$$

Milgrom (1983)

JO, Russo, 2402.19495

■

$$ds^2 = - \left(1 - \frac{2MG_N}{r} - \gamma_1 r - \gamma_2 r^2 \right) dt^2 + \left(1 - \frac{2MG_N}{r} - \gamma_1 r - \gamma_2 r^2 \right)^{-1} dr^2 + r^2 d\Omega^2$$

Riegert (1984)

Mannheim, Kazanas (1989)

Challenges

Tensions?

CPTP, normalisable

Local

Renormalisable in matter sector?

Covariant

Anomalous heating



