Dynamically Assisted Tunneling

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DRESDEN concept
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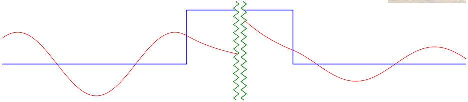
Motivation

S. Coleman: *"Every child knows..."*

$$P \sim \exp\left\{-\frac{2}{\hbar}\int dx\,\sqrt{2m[V(x)-E]}\right\}$$

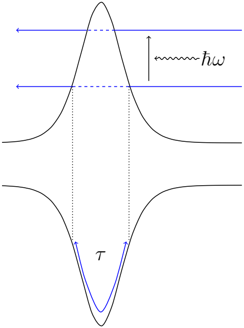
Question: $V(x) \to V(t,x)$?



Here: $V(x)$ plus field $\mathfrak{E}(t)$

- pre-acceleration
- potential deformation
- energy mixing → Franz-Keldysh effect
  $E \to E + \hbar\omega$ (Floquet ansatz)

  W. Franz, Z. Naturforsch. **13** A, 484 (1958);
  L. V. Keldysh, Sov. Phys. JETP **34**, 788 (1958).
- what else?



Adiabatic versus non-adiabatic:

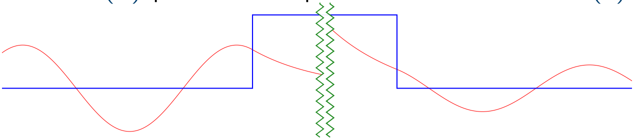
Büttiker-Landauer "traversal" time

$$\mathfrak{T} = \sqrt{m}\int dx\,/\sqrt{2[V(x)-E]}$$

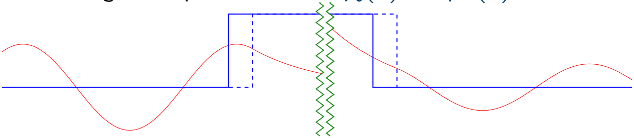
M. Büttiker and R. Landauer, PRL **49**, 1739 (1982).

Dynamically Assisted Tunneling

Potential barrier $V(x)$ plus time-dependent electric field $\mathfrak{E}(t)$



Kramers-Henneberger displacement $m\ddot{\chi}(t) = q\mathfrak{E}(t)$



Energy mixing $\chi(t + i\mathfrak{T})$ displacement ("pushing out") $\chi(t)$

$$\psi_{\rm tra}(E) \approx \psi_E^0 \int \frac{dt}{2\pi}\, e^{i(E-E_{\rm in})t-\sqrt{2mV_0}[\chi(t+i\mathfrak{T})-\chi(t)]}$$

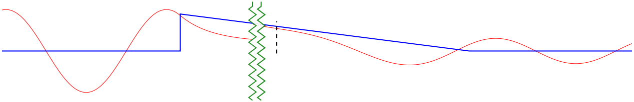
Low-energy + opaque-barrier approximation

C. Kohlfürst, F. Queisser and R.S., Phys. Rev. Research **3**, 033153 (2021).

→ instanton picture ✓

Triangular Barrier → Quantum Ratchets

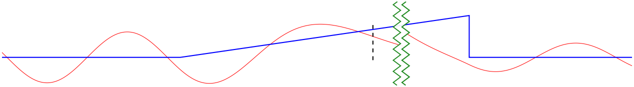
Steep incidence



Mainly energy mixing at front end

$$\psi_{\rm tra}(E) \approx \psi_E^0 \int \frac{dt}{2\pi}\, e^{i(E-E_{\rm in})t-\sqrt{2mV_0}\chi(t+i\mathfrak{T})}$$

Gradual incidence



Mainly displacement at rear end

$$\psi_{\rm tra}(E) \approx \psi_E^0 \int \frac{dt}{2\pi}\, e^{i(E-E_{\rm in})t+\sqrt{2mV_0}\chi(t)}$$

→ quantum ratchets

Rough Scaling Analysis

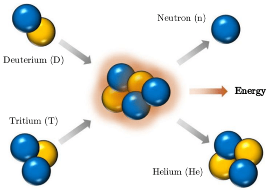
Büttiker-Landauer "traversal" time

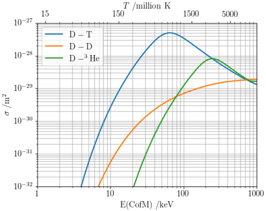
$$\mathfrak{T} = \int\limits_{x_{\text{in}}^E}^{x_{\text{out}}^E} dx \sqrt{\frac{m}{2[V(x)-E]}} \sim \mathcal{O}(L^2 m/\hbar) \sim 1/\omega$$

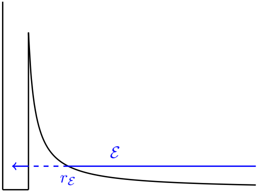
Length	System	Energy	Field Strength
μm	optical lattices	peV	n.a.
nm	solids	meV	10^5 V/m
pm	atoms	eV	10^{10} V/m
	nuclear fusion	keV	10^{16} V/m
fm	α-decay	MeV	10^{18} V/m

Relativistic: Sauter-Schwinger $\mathfrak{E}_{\text{crit}} = m_e c^3/(q_e \hbar) \approx 1.3 \times 10^{18}$ V/m

Dynamically Assisted Nuclear Fusion







F. Queisser and R.S., Phys. Rev. C **100**, 041601(R) (2019).

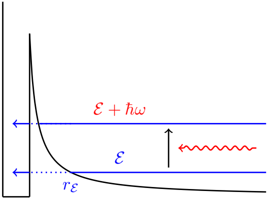
$${}^{2}_{1}\mathrm{D} + {}^{3}_{1}\mathrm{T} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} + 17.6\ \mathrm{MeV}$$

$${}^{1}_{1}\mathrm{p} + {}^{11}_{5}\mathrm{B} \rightarrow 3 \times {}^{4}_{2}\mathrm{He} + 8.7\ \mathrm{MeV}$$

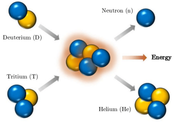
Assistance by XFEL field/pulse

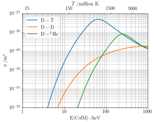
$$A_x(t) = A_0 / \cosh^2(\omega t)$$

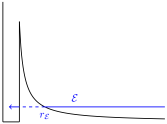
Exponent (instanton) vs pre-factor...

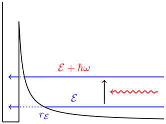


Analytical Model









Two-body Lagrangian with Coulomb (+nuclear) field and XFEL

$$L_{12}=\frac{m_1}{2}\dot{\boldsymbol{r}}_1^2+\frac{m_2}{2}\dot{\boldsymbol{r}}_2^2-V(|\boldsymbol{r}_1-\boldsymbol{r}_2|)+(q_1\dot{\boldsymbol{r}}_1+q_2\dot{\boldsymbol{r}}_2)\cdot\boldsymbol{A}(t)$$

Center-of-mass and relative coordinates with reduced mass

$$L=\frac{m}{2}\dot{\boldsymbol{r}}_-^2-V(|\boldsymbol{r}_-|)+q_{\rm eff}\,\dot{\boldsymbol{r}}_-\cdot\boldsymbol{A}(t)$$

Approximate scaling symmetry → dimension-less parameters

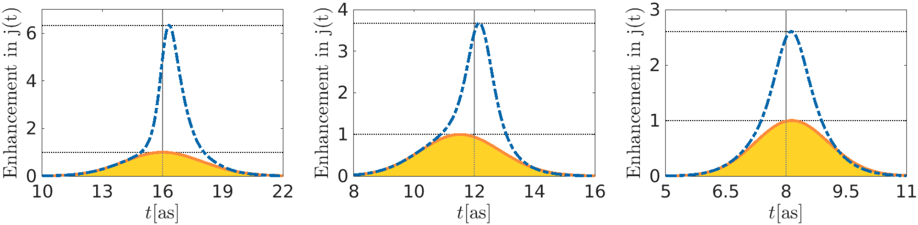
$$\eta=2mEr_E^2=\frac{2m}{E}\left(\frac{q_1q_2}{4\pi\epsilon_0}\right)^2\,,\qquad\zeta=\frac{q_{\rm eff}A}{m\omega r_E}=\frac{q_{\rm eff}A}{mc}\,\frac{E}{\omega}\,\frac{4\pi\epsilon_0c}{q_1q_2}$$

WKB tunneling exponent $\mathcal{P}\sim\exp\{-\pi\sqrt{\eta}\}$

Scaling $E_{\rm p+B}\leftrightarrow 20E_{\rm D+T}$

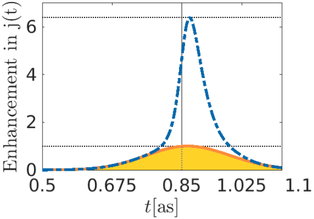
Numerical Simulations

1D-Schrödinger solver for D+T fusion with $\omega = 1$ keV and 10^{16} V/m



Initial kinetic energy: 2 keV, 4 keV and 8 keV

Comparison: p+B fusion with
$E = 38$ keV and pulse with
$\omega = 19$ keV and 28×10^{16} V/m
→ scaling behavior



C. Kohlfürst, F. Queisser and R.S., Phys. Rev. Research **3**, 033153 (2021).

Field of α-particles?

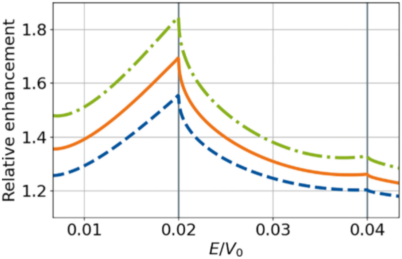
Resonance Effects

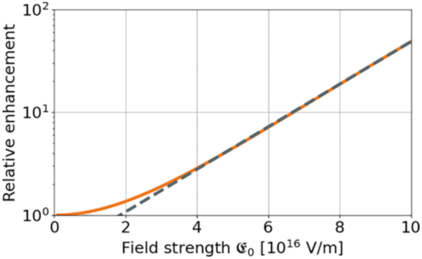
$A_x(t) = A_0 \cos(\omega t) \to$ resonances at $E = n\omega$ (Floquet channels)

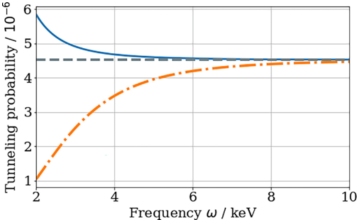
D. Ryndyk, C. Kohlfürst, F. Queisser, R.S., Phys. Rev. Research **6**, 023056 (2024).

Box potential

Coulomb potential







Time-averaged potential approximation?

Sauter-Schwinger Effect

F. Sauter, Z. Phys. **69**, 742 (1931); J. S. Schwinger, Phys. Rev. **82**, 664 (1951);…

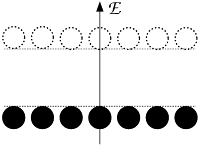
Schrödinger equation (non-relativistic)

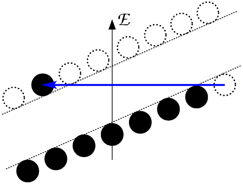
$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \rightsquigarrow E = \frac{p^2}{2m} + V$$

Dirac equation (relativistic)

$$\gamma^\mu\left(i\hbar\partial_\mu + qA_\mu\right)\Psi = mc\Psi \rightsquigarrow E = V \pm \sqrt{c^2p^2 + m^2c^4}$$

Positive and negative energy levels → Dirac sea → holes = positrons





Electric field: tilt $V(x) = q\mathfrak{E}x$

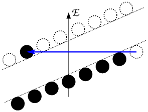
→ tunneling from Dirac sea

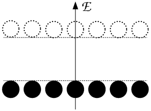
$$\mathfrak{E}_{\text{crit}} = mc^3/(q\hbar) \approx 1.3 \times 10^{18}\text{V/m}$$

Matter from Light

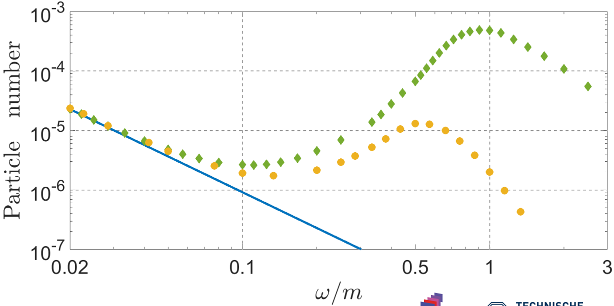
C.Kohlfürst, N.Ahmadiniaz, J.Oertel, R.S., Phys. Rev. Lett. **129**, 241801 (2022).

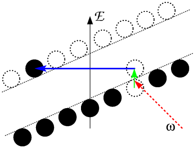
Sauter-Schwinger (non-perturbative) Breit-Wheeler (perturbative)





Colliding laser pulses (Maxwell equations ✓ transversal fields ✓)





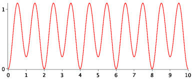
instanton ✓

pre-factor ?

WKB ✓

Quantum Simulators

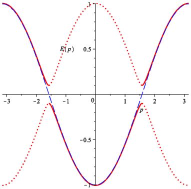
N. Szpak, R. S., Phys. Rev. A **84** (R), 050101 (2011); New J. Phys. **14**, 035001 (2012).



Fermions in bi-chromatic 1D-lattice (e.g., optical)

Continuum limit → Dirac equation with $m_{\text{eff}}(t)$!!!

$$\gamma^{\mu}\left(i\hbar\partial_{\mu}+qA_{\mu}^{\text{eff}}\right)\Psi=m_{\text{eff}}\,c_{\text{eff}}\,\Psi$$



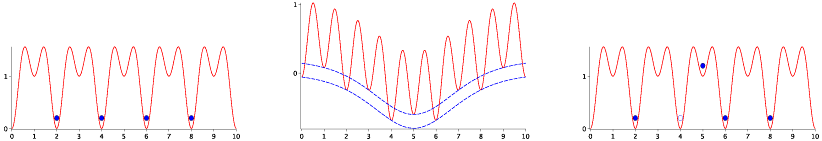
"*The same equations have the same solutions.*"

Lower band filled → Dirac sea

Lattice deformation → $\mathfrak{E}_{\text{eff}}(t)$

Lattice gap → $2m_{\text{eff}}c_{\text{eff}}^{2}\lll 2m_{e}c^{2}$

Hopping rate $\hbar J\ell_{\text{lattice}}\to c_{\text{eff}}\lll c$

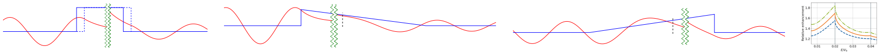


Cf. M.F.Linder, A.Lorke, R.S., Phys. Rev. B **97**, 035203 (2018).

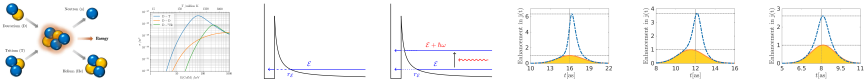
Summary and Outlook

Dynamically assisted tunneling

C. Kohlfürst, F. Queisser and R.S., Phys. Rev. Research **3**, 033153 (2021);
D. Ryndyk, C. Kohlfürst, F. Queisser, R.S., Phys. Rev. Research **6**, 023056 (2024).



Dynamically assisted nuclear fusion F. Queisser and R.S., Phys. Rev. C **100**, 041601(R) (2019).



Dynamically assisted Sauter-Schwinger effect → quantum simulators

C.Kohlfürst, N.Ahmadiniaz, J.Oertel, R.S., Phys. Rev. Lett. **129**, 241801 (2022).