Energy transfer between gravitational waves and quantum matter

Ralf Schützhold[1,2]

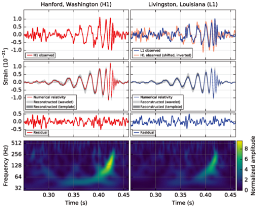
[1] *Helmholtz-Zentrum Dresden-Rossendorf*
[2] *Institut für Theoretische Physik, Technische Universität Dresden*

Los Alamos, June 6th 2024

Introduction

Gravitational wave in z-direction, fixed (+) polarization, TT gauge

$$ds^2 = dt^2 - [1 + h(t,z)]\, dx^2 - [1 - h(t,z)]\, dy^2 - dz^2 + \mathcal{O}(h^2)$$



B. P. Abbott *et al* (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. **116**, 061102 (2016)

- type-I: *during* gravitational wave, e.g., LIGO
- type-II: *after* gravitational wave, e.g., Weber bars

Quantum matter (e.g., superfluid helium) as resonant mass antenna?

E.g., *Detecting continuous gravitational waves with superfluid* ^{4}*He*, S. Singh *et al*, New J. Phys. **19**, 073023 (2017); *Phonon creation by gravitational waves*, C. Sabín *et al*, New J. Phys. **16**, 085003 (2014), etc.

Matter waves interferometers: type-I

Hamiltonian

Non-relativistic Hamiltonian, first in flat space-time

$$\hat{H}_0 = \int \mathrm{d}^3 r \left[\frac{1}{2m}(\nabla\hat{\Psi}^\dagger)\cdot(\nabla\hat{\Psi}) + V_0(\boldsymbol{r})\hat{\Psi}^\dagger\hat{\Psi}\right]$$
$$+\frac{1}{2}\int \mathrm{d}^3 r\,\mathrm{d}^3 r'\,\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}^\dagger(\boldsymbol{r}')W(\boldsymbol{r},\boldsymbol{r}')\hat{\Psi}(\boldsymbol{r}')\hat{\Psi}(\boldsymbol{r})$$

Impact of gravitational wave $h(t,z) \approx h(t)$

- kinetic term $(\nabla\hat{\Psi}^\dagger)\cdot(\nabla\hat{\Psi})$ is replaced by $-(\partial_i\hat{\Psi}^\dagger)g^{ij}(\partial_j\hat{\Psi})$

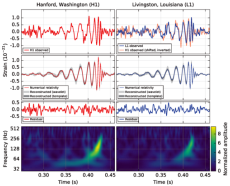
  S. Fagnocchi *et al*, New J. Phys. **12**, 095012 (2010)
- potential $V_0(\boldsymbol{r}) \to V(t,\boldsymbol{r})$ from microscopic description
- interaction W (short-range, isotropic) is approximately unaffected

$$\hat{H}_1 = h\int \mathrm{d}^3 r \left[\frac{\partial V}{\partial h}\hat{\Psi}^\dagger\hat{\Psi} + \frac{(\partial_y\hat{\Psi}^\dagger)(\partial_y\hat{\Psi}) - (\partial_x\hat{\Psi}^\dagger)(\partial_x\hat{\Psi})}{2m}\right] + \mathcal{O}(h^2)$$

Interaction of a Bose-Einstein condensate with a gravitational wave, R. Schützhold, Phys. Rev. D **98**, 105019 (2018)

TECHNISCHE UNIVERSITÄT DRESDEN   DRESDEN concept   HZDR

Energy Transfer



Heisenberg picture

$$\frac{d\hat{H}}{dt} = \left(\frac{\partial \hat{H}}{\partial t}\right)_{\rm expl} = \frac{\partial \hat{H}}{\partial h}\dot{h} + \mathcal{O}(h^2)$$

$$= \dot{h}\int d^3r \left[\frac{\partial V}{\partial h}\hat{\Psi}^\dagger\hat{\Psi} + \frac{(\partial_y\hat{\Psi}^\dagger)(\partial_y\hat{\Psi}) - (\partial_x\hat{\Psi}^\dagger)(\partial_x\hat{\Psi})}{2m}\right] + \mathcal{O}(h^2)$$

Expectation value to first order in h

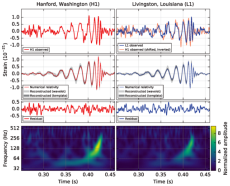
$$\dot{E} = \dot{h}\int d^3r \left\langle \frac{\partial V}{\partial h}\hat{\Psi}^\dagger\hat{\Psi} + \frac{(\partial_y\hat{\Psi}^\dagger)(\partial_y\hat{\Psi}) - (\partial_x\hat{\Psi}^\dagger)(\partial_x\hat{\Psi})}{2m}\right\rangle_0 + \mathcal{O}(h^2)$$

Non-stationary states $\langle\dots\rangle_0$ required for energy transfer

$$\dot{E} \leq |\dot{h}|_{\max}\left(\left|\frac{\partial V}{\partial h}\right|_{\max}\langle\hat{N}\rangle + \langle\hat{E}_{\rm kin}\rangle_{\max}\right)$$

Rigorous bound!

Example Values



$$\dot{E} \leq |\dot{h}|_{\max} \left(\left| \frac{\partial V}{\partial h} \right|_{\max} \langle \hat{N} \rangle + \langle \hat{E}_{\text{kin}} \rangle_{\max} \right)$$

- gravitational wave amplitude $h = \mathcal{O}(10^{-22})$
- gravitational wave frequency $\omega = \mathcal{O}(\text{kHz})$
- interaction time T of 100 cycles $\omega T = \mathcal{O}(10^2)$ (resonance)
- energy shift $\Delta E = \mathcal{O}(h\omega T E_{\text{kin}})$ of one excitation quantum $\mathcal{O}(\hbar\omega)$

$\to$ required kinetic energy $\langle \hat{E}_{\text{kin}} \rangle_{\max} = \mathcal{O}(10^8 \text{ eV}) = \mathcal{O}(10^{-11} \text{ J})$

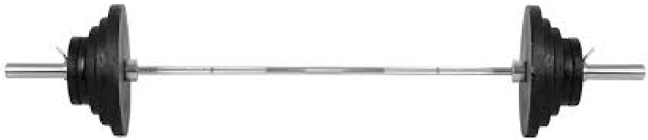
Bose-Einstein condensate in optical trap C. Sabín *et al*, New J. Phys. **16**, 085003 (2014)

- recoil energy $E_R = k^2/(2m) = \mathcal{O}(\mu\text{K})$
- assume large atom number $N = \mathcal{O}(10^9)$
- energy per particle $\langle \hat{E}_{\text{kin}} \rangle_{\max}/N = \mathcal{O}(0.1 \text{ eV}) = \mathcal{O}(10^3 \text{ K})$

Detecting one excitation quantum $\hbar\omega$ on top of huge background...

TECHNISCHE UNIVERSITÄT DRESDEN DRESDEN concept HZDR

Resonantly Rotating and Vibrating Barbell



Reaching $E_{\text{kin}} = \mathcal{O}(10^{-11}\text{ J})$ is easy, but detecting small changes...

$$\Delta E_{\text{vib}} = \mathcal{O}\left(h\omega T\sqrt{E_{\text{vib}}E_{\text{rot}}}\right) \quad \text{with} \quad \omega_{\text{vib}} = |\omega \pm 2\omega_{\text{rot}}|$$

Idea: large rotational E_{rot} but small vibrational energy E_{vib}

- resonant energy transfer ΔE_{vib} of one excitation quantum $\hbar\omega_{\text{vib}}$
- initial quantum state of vibrational mode: ten excitation quanta
- required rotational energy $E_{\text{rot}} = 10^8$ J
- mass $m = \mathcal{O}(100\text{ kg})$ length $R = \mathcal{O}(\text{m})$ frequency $\omega_{\text{rot}} = \mathcal{O}(\text{kHz})$

Alternative: two barbells with scissor motion

TECHNISCHE UNIVERSITÄT DRESDEN · DRESDEN concept · HZDR
Member of the Helmholtz Association
R. Schützhold | Theoretical Physics | http://www.hzdr.de

Change of Potential

Bose-Einstein condensate in optical trap C. Sabín *et al*, New J. Phys. **16**, 085003 (2014)

Frequency of electromagnetic waves from dispersion relation

$$\Omega^2 = -g^{ij}K_iK_j = [1-h]K_x^2 + [1+h]K_y^2 + K_z^2$$

Amplitude from Maxwell equations $\nabla_\mu F^{\mu\nu} = \partial_\mu(\sqrt{-g}F^{\mu\nu})/\sqrt{-g} = 0$

$$\left(\partial_t^2 - \partial_x[1-h]\partial_x - \partial_y[1+h]\partial_y\right) A_z(t,x,y) = 0$$

Conserved Wronskian $W = A_z^*\dot{A}_z - \dot{A}_z^*A_z \approx -2i\Omega|A_z^2|$

$$\left(\partial_t[1-h]\partial_t - \partial_y^2 - \partial_z[1-h]\partial_z\right) A_x = 0$$

Conserved Wronskian $W = [1-h](A_x^*\dot{A}_x - \dot{A}_x^*A_x) \approx -2i[1-h]\Omega|A_x^2|$

In general $W = A_i^* g^{ij}\dot{A}_j - \dot{A}_i^* g^{ij} A_j \to$ direction & polarization

Interaction with atoms $\propto 1/(\Omega^2 - \Omega_{\text{res}}^2)$ resonance effects...

Atomic Levels Ω_{res}

Undisturbed Hamiltonian with Coulomb potential $V(\hat{r}) = -q^2/(4\pi\hat{r})$

$$\hat{H}_0 = \frac{\hat{\boldsymbol{p}}^2}{2m} + V(\hat{r})$$

Perturbation Hamiltonian due to gravitational wave

$$\hat{H}_1 = h\left[\frac{\hat{p}_y^2 - \hat{p}_x^2}{2m} + q^2\frac{\hat{x}^2 - \hat{y}^2}{8\pi\hat{r}^3}\right] = \hat{H}_1^{\text{kin}} + \hat{H}_1^{\text{pot}}$$

Shift of kinetic and potential energy cancel to first order (as expected)

$$\langle 2\mathrm{p}_x | \hat{H}_1^{\text{kin}} | 2\mathrm{p}_x \rangle = -h\frac{q^2}{80\pi a_{\mathrm{B}}} = -\langle 2\mathrm{p}_x | \hat{H}_1^{\text{pot}} | 2\mathrm{p}_x \rangle$$

Matrix elements can be non-zero

$$\langle 1\mathrm{s} | \hat{H}_1 \left| 3\mathrm{d}_{x^2-y^2} \right\rangle = h\frac{q^2}{128\pi a_{\mathrm{B}}}$$

$\rightarrow$ deformation of eigenstates

Bose-Einstein Condensates

Combine all factors $1 + \zeta h$ from various sources...

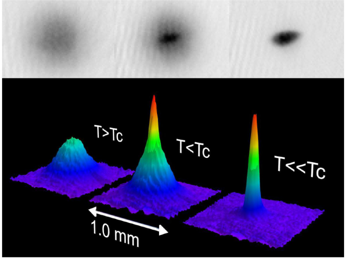
Gaussian approximation

$$V(t, \boldsymbol{r}) = \boldsymbol{r} \cdot (\boldsymbol{M}_0 + h\boldsymbol{M}_1) \cdot \boldsymbol{r} + h\boldsymbol{F}_1 \cdot \boldsymbol{r} + hV_1$$

Order-of magnitude estimate

- shape deformation $\boldsymbol{M}_1 = \mathcal{O}(E_\mathrm{R}/\lambda^2)$
- shift in position $\boldsymbol{F}_1 = \mathcal{O}(E_\mathrm{R}/\lambda)$ (for asymmetric scenarios)
- shift in energy $V_1 = \mathcal{O}(E_\mathrm{R})$ (does not contribute)

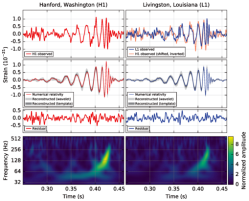
$\rightarrow$ analogous order of magnitude as kinetic energy



TECHNISCHE UNIVERSITÄT DRESDEN
DRESDEN concept
HZDR

Fidelity

So far: energy transfer $\to$ type-II detection scheme



In general: detection only possible if quantum states differ $\to$ fidelity

$$\langle\psi|\mathcal{T}\exp\left\{-\frac{i}{\hbar}\int dt\,\hat{H}_1(t)\right\}|\psi\rangle = 1 - \frac{i}{\hbar}\int dt\langle\hat{H}_1(t)\rangle_0 + \mathcal{O}(h^2)$$

Very analogous estimate (apart from overall factor ω)

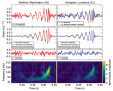
$$\Delta\varphi \leq T|h|_{\max}\left(\left|\frac{\partial V}{\partial h}\right|_{\max}\langle\hat{N}\rangle + \langle\hat{E}_{\text{kin}}\rangle_{\max}\right) \stackrel{?}{=} \mathcal{O}(1)$$

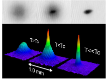
Trapped Bose-Einstein condensates versus matter-wave interferometers

$\to$ length scales etc.

Summary & Outlook

Energy transfer between gravitational waves and quantum matter,
J. Gräfe, F. Adamietz, R. Schützhold, Phys. Rev. D **108**, 064056 (2023)





- non-stationary states required (resonance) to first order in h
- rigorous bound $\dot{E} \leq |\dot{h}|_{\text{max}}(|\partial V/\partial h|_{\text{max}}\langle\hat{N}\rangle + \langle\hat{E}_{\text{kin}}\rangle_{\text{max}})$ for type-II
- analogously $\Delta\varphi \leq T|h|_{\text{max}}(|\partial V/\partial h|_{\text{max}}\langle\hat{N}\rangle + \langle\hat{E}_{\text{kin}}\rangle_{\text{max}})$ for type-I
- Bose-Einstein condensates ??? rotating + vibrating barbell ?

Outlook

- in phase: absorb energy from gravitational wave
- opposite phase: stimulated emission of gravitons