Catalytic enhancements in the performance of the microscopic two-stroke engine.

Tanmoy Biswas (Theoretical Division, LANL)

QMCOSMOS Workshop
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arXiv: 2402. 10384 , LA-UR 24-20752
LA-UR-24-25471

Motivation.

1) Understanding thermodynamics at the microscopic regime.

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2) Size vs efficiency trade-off.

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1) Understanding thermodynamics at the microscopic regime.

2) Size vs efficiency trade-off.

3) Quantum computing : Resetting of qubits.

Marcin Łobejko

Paweł Mazurek

Michał Horodecki

International Centre for Theory of Quantum Technologies (ICTQT)

University of Gdańsk

Outline of the talk.

1) Two-stroke heat engine in the microscopic regime.
   A) Without Catalyst.
   B) With Catalyst.

2) Thermodynamic framework.

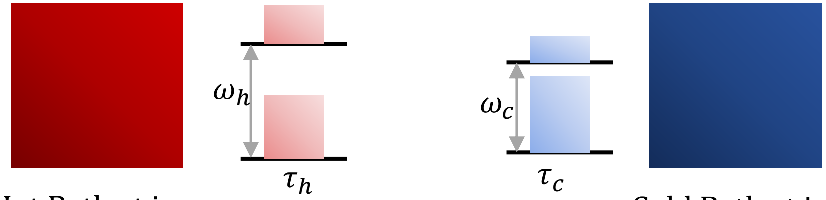
3) Enhancements in the efficiency due to the catalyst.

4) Conclusion

Two-stroke heat engine in the microscopic regime.

Initial state

Without catalyst



Hot Bath at inverse temperature (β_h)

Cold Bath at inverse temperature (β_c)

$$\tau_h \otimes \tau_c$$

$$\tau_h = \frac{1}{(1+e^{-\beta_h \omega_h})}\begin{pmatrix} 1 & 0 \\ 0 & e^{-\beta_h \omega_h} \end{pmatrix} \; ; \; \tau_c = \frac{1}{(1+e^{-\beta_c \omega_c})}\begin{pmatrix} 1 & 0 \\ 0 & e^{-\beta_c \omega_c} \end{pmatrix}.$$

Two-stroke heat engine in the microscopic regime.

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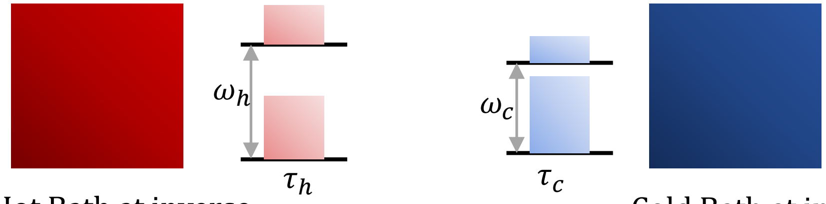

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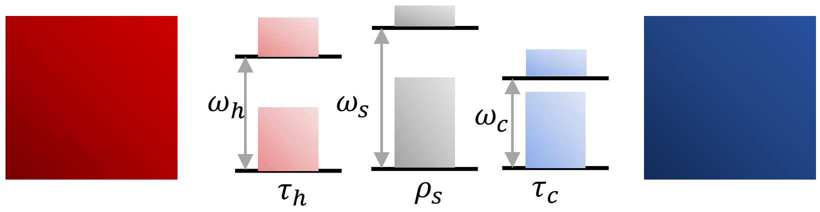
With catalyst



Hot Bath at inverse temperature (β_h)

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$$\rho_s \otimes \tau_h \otimes \tau_c$$

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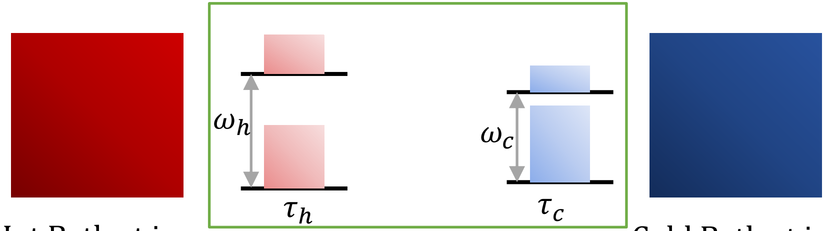
Two-stroke heat engine in the microscopic regime.

Work Stroke

Without catalyst



Hot Bath at inverse temperature (β_h)

Unitary (U)

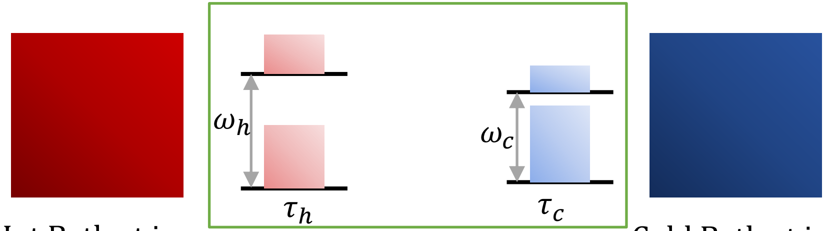
Cold Bath at inverse temperature (β_c)

$$U(\tau_h \otimes \tau_c)U^\dagger$$

Two-stroke heat engine in the microscopic regime.

Work Stroke

Without catalyst



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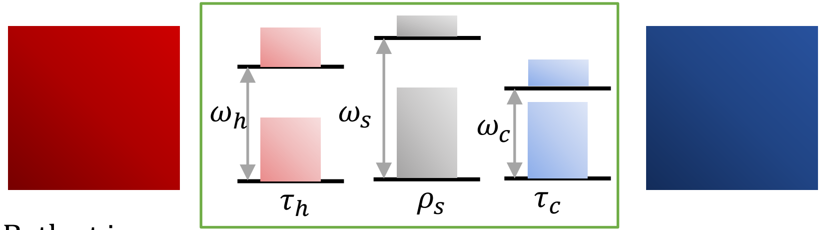


Hot Bath at inverse temperature (β_h)

Unitary (U)

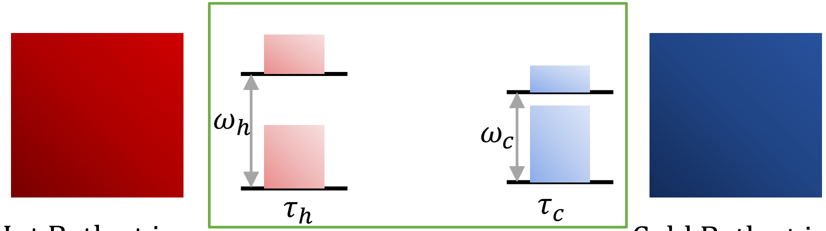
$$U(\rho_s \otimes \tau_h \otimes \tau_c)U^\dagger$$

Catalysis condition: $Tr_{h,c} U(\rho_s \otimes \tau_h \otimes \tau_c)U^\dagger = \rho_s$

Two-stroke heat engine in the microscopic regime.

Work Stroke

Without catalyst



Hot Bath at inverse temperature (β_h)

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Cold Bath at inverse temperature (β_c)

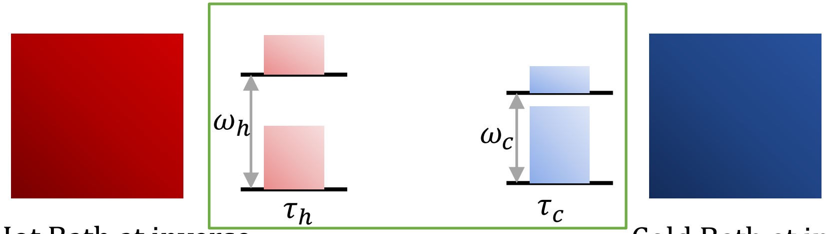
$$U(\tau_h \otimes \tau_c)U^\dagger$$

$$W = Tr[(H_h + H_c)(\tau_h \otimes \tau_c - U(\tau_h \otimes \tau_c)U^\dagger)]$$

Two-stroke heat engine in the microscopic regime.

Work Stroke

Without catalyst



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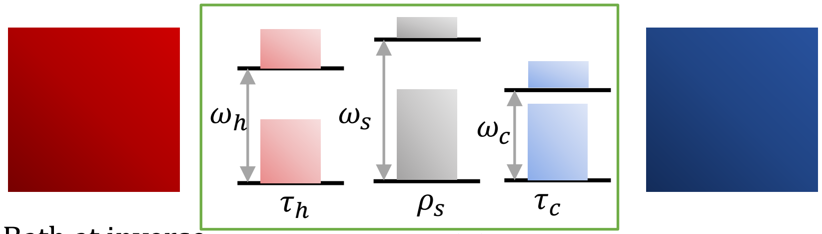
With catalyst



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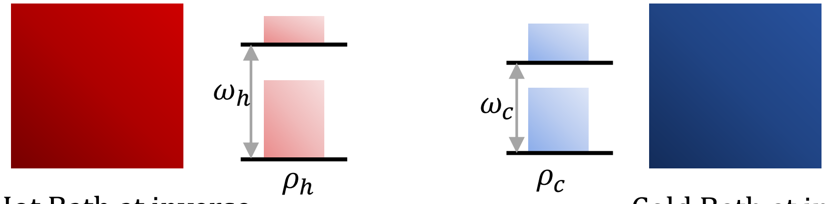
Two-stroke heat engine in the microscopic regime.

Work Stroke

Without catalyst



Hot Bath at inverse temperature (β_h)

Cold Bath at inverse temperature (β_c)

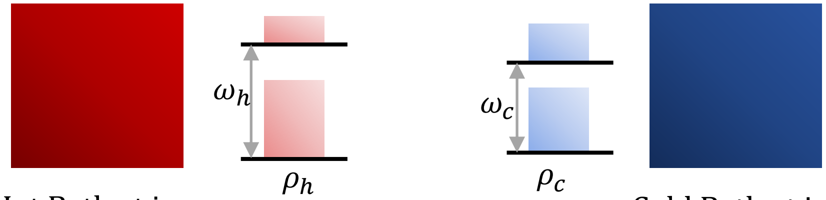
Final state of hot qubit $Tr_h(U(\tau_h \otimes \tau_c)U^\dagger) = \rho_h$

Final state of cold qubit $Tr_c(U(\tau_h \otimes \tau_c)U^\dagger) = \rho_c$

Two-stroke heat engine in the microscopic regime.

Work Stroke

Without catalyst



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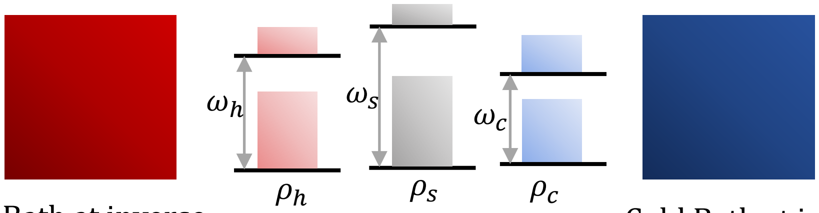
With catalyst



Hot Bath at inverse temperature (β_h)

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Final state of hot qubit $Tr_{s,h}(U(\rho_s \otimes \tau_h \otimes \tau_c)U^\dagger) = \rho_h$

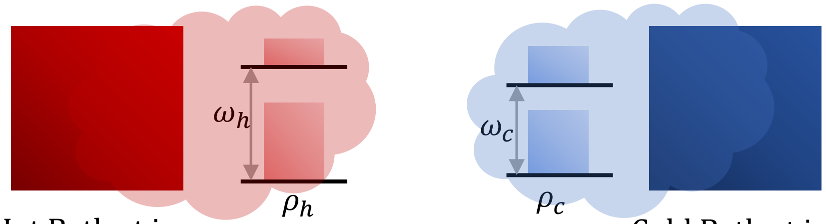
Final state of cold qubit $Tr_{s,c}(U(\rho_s \otimes \tau_h \otimes \tau_c)U^\dagger) = \rho_c$

Final state of the catalyst $Tr_{h,c}(U(\rho_s \otimes \tau_h \otimes \tau_c)U^\dagger) = \rho_s$

Two-stroke heat engine in the microscopic regime.

Heat Stroke

Without catalyst



Hot Bath at inverse temperature (β_h)

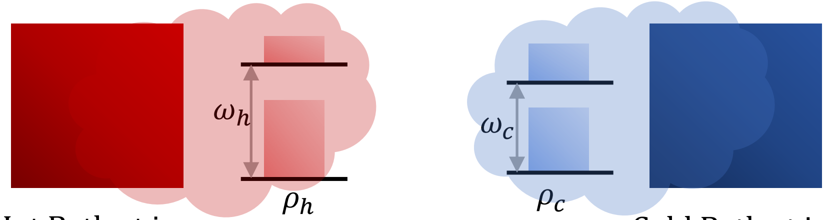
Cold Bath at inverse temperature (β_c)

Rethermalization with respective baths

Two-stroke heat engine in the microscopic regime.

Heat Stroke

Without catalyst



Hot Bath at inverse temperature (β_h)

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Rethermalization with respective baths

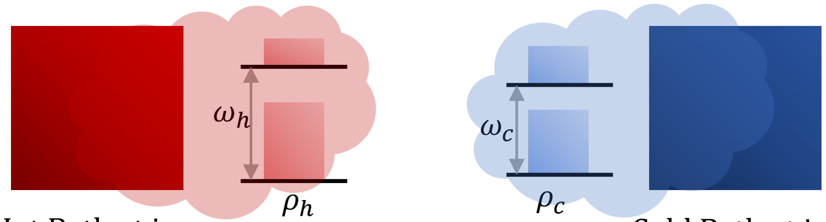
Heat withdrawn from hot bath : $Q_h = Tr[H_h(\tau_h - \rho_h)]$

Heat discharged into cold bath : $Q_c = Tr[H_c(\tau_c - \rho_c)]$

Two-stroke heat engine in the microscopic regime.

Heat Stroke

Without catalyst



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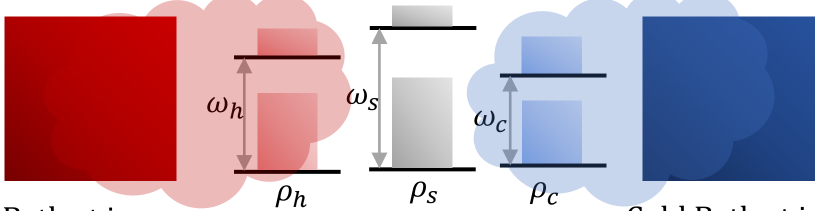

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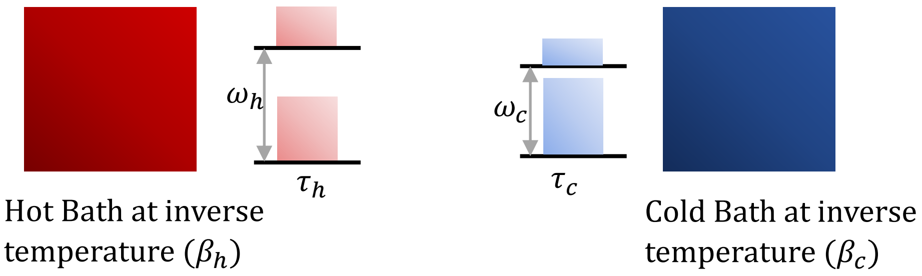
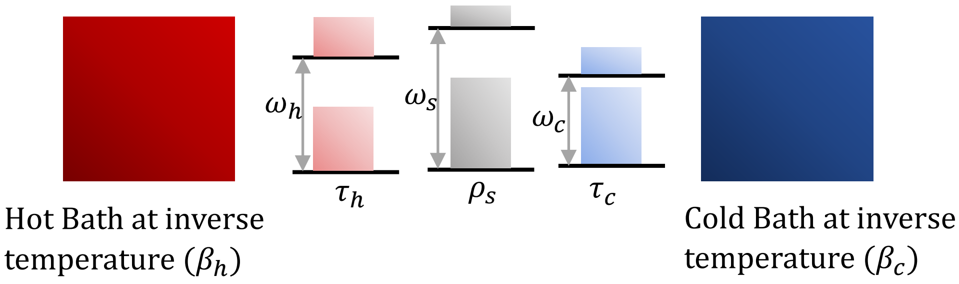
Two-stroke heat engine in the microscopic regime.

Closing the cycle

Without catalyst



With catalyst



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Thermodynamic framework.

Manifestation of First Law:

$$W = Q_h + Q_c$$

Heat withdrawn from hot bath := Q_h

Heat discharged into cold bath:= Q_c

Work produced by the engine := W

Thermodynamic framework.

Manifestation of First Law:

$$W = Q_h + Q_c$$

Heat withdrawn from hot bath := Q_h

Heat discharged into cold bath:= Q_c

Work produced by the engine := W

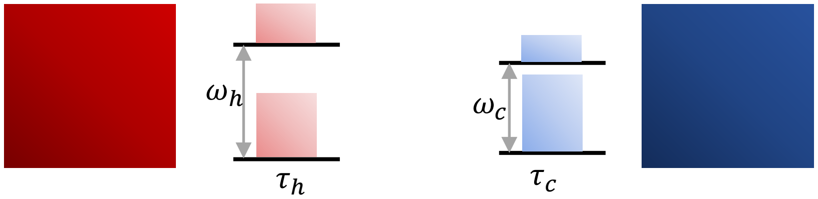
Manifestation of Second Law:

Efficiency := η

$$\eta \coloneqq \frac{W}{Q_h} = 1 + \frac{Q_c}{Q_h} \leq 1 - \frac{\beta_h}{\beta_c} = \eta_{Carnot}$$

Optimal performance without catalyst.

For a fixed value of ω_h and ω_c



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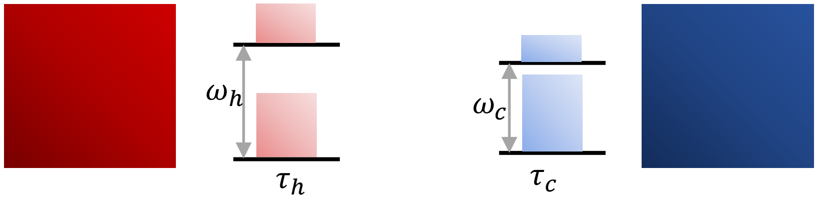
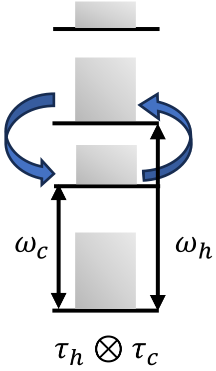


For a fixed value of ω_h and ω_c

$$\text{Optimal Work} = \frac{1}{(1+a_h)(1+a_c)}(a_h - a_c)(\omega_h - \omega_c)$$

$$a_c = e^{-\beta_c \omega_c}$$

$$a_h = e^{-\beta_h \omega_h}$$



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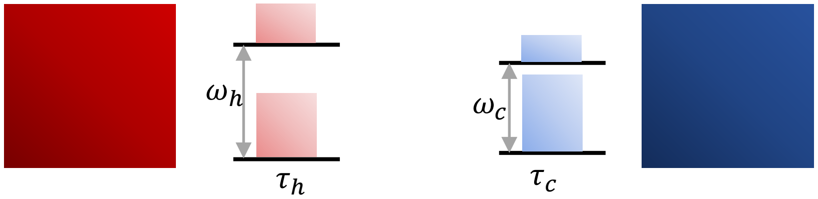

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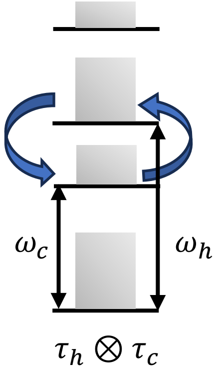
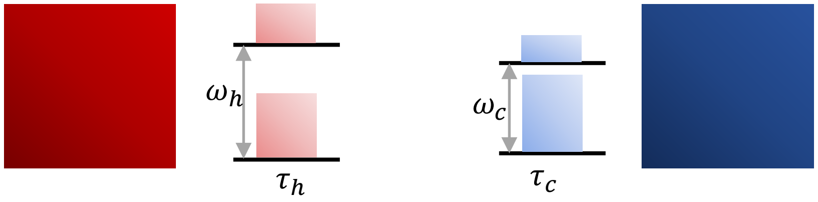


Optimal Efficiency $= 1 - \frac{\omega_c}{\omega_h}$. (Otto Efficiency)

Optimal performance without catalyst.

For a fixed value of ω_h and ω_c



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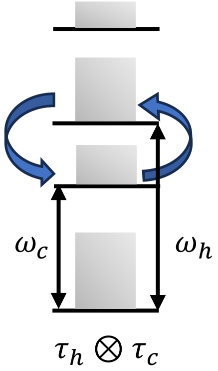
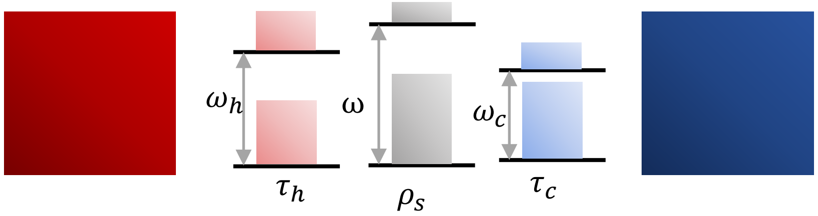


$$\text{Optimal Efficiency} = 1 - \frac{\omega_c}{\omega_h}. \text{ (Otto Efficiency)}$$

When $\frac{\omega_c}{\omega_h} = \frac{\beta_h}{\beta_c}$ then $a_h = a_c$. This implies optimal work is 0

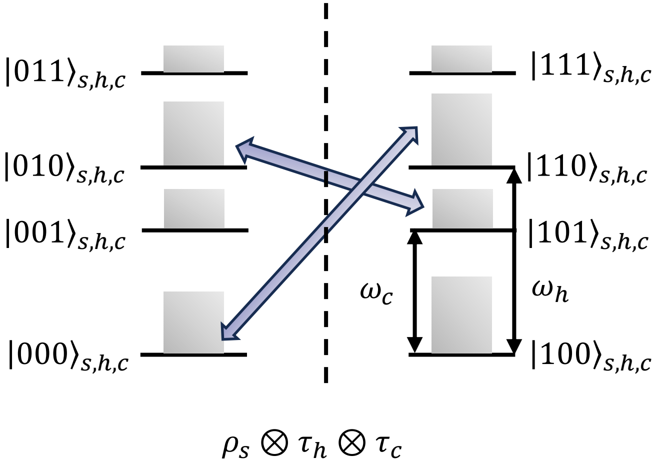
Catalytic enhancement

For a fixed value of ω_h and ω_c



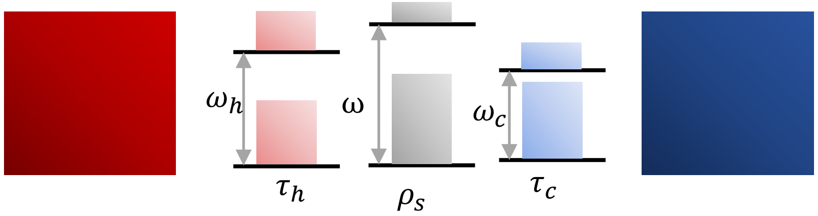
Efficiency: $\eta_2 = 1 - \frac{\omega_c}{2\omega_h} > 1 - \frac{\omega_c}{\omega_h}$

$=$ Optimal efficiency without catalyst



Catalysis condition: $Tr_{h,c} U(\rho_s \otimes \tau_h \otimes \tau_c) U^\dagger = \rho_s$

Catalytic enhancement



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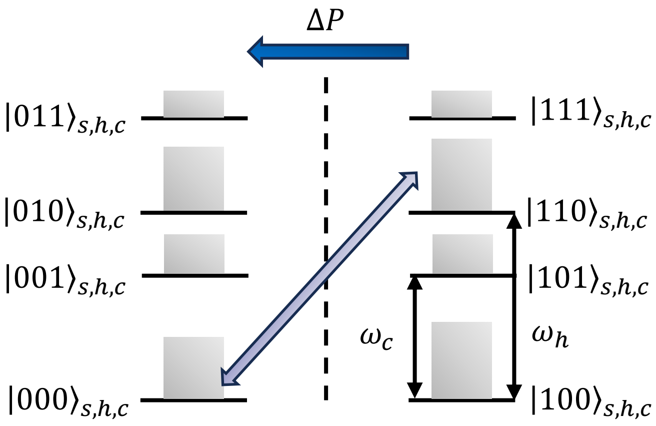

$\rho_s \otimes \tau_h \otimes \tau_c$

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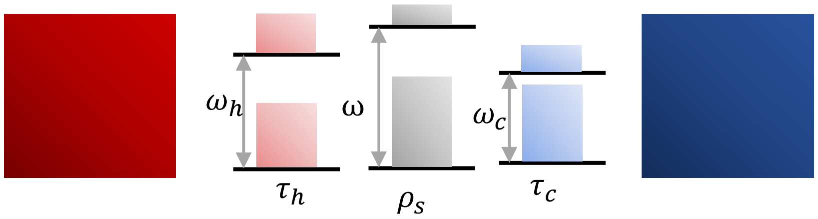

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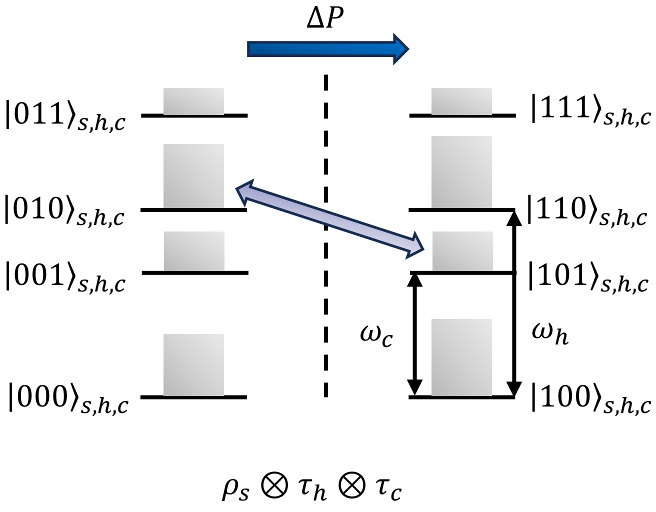

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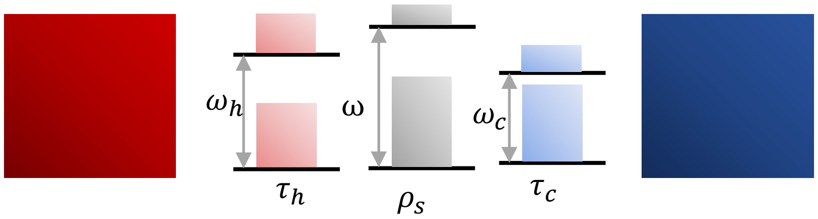

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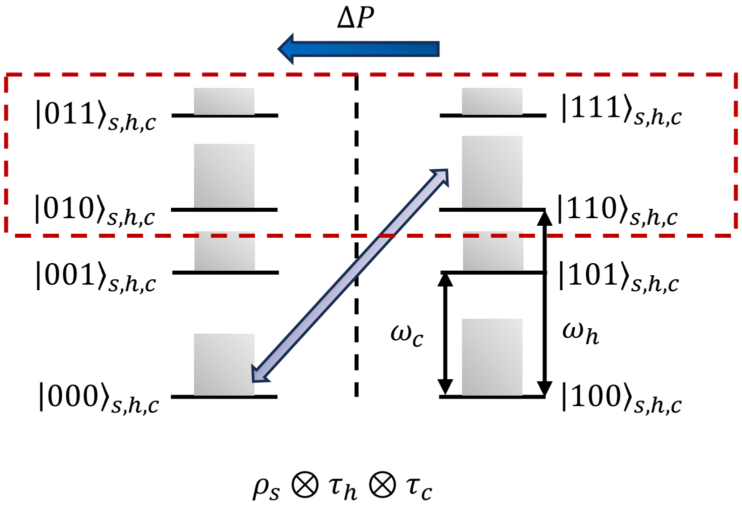

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Catalytic enhancement

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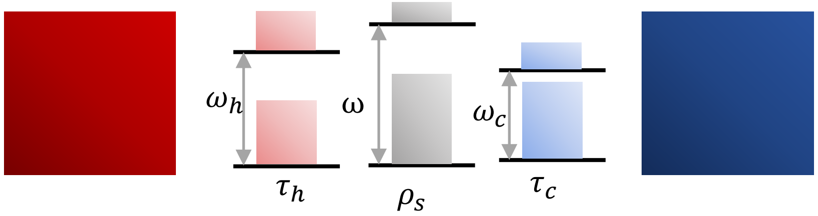

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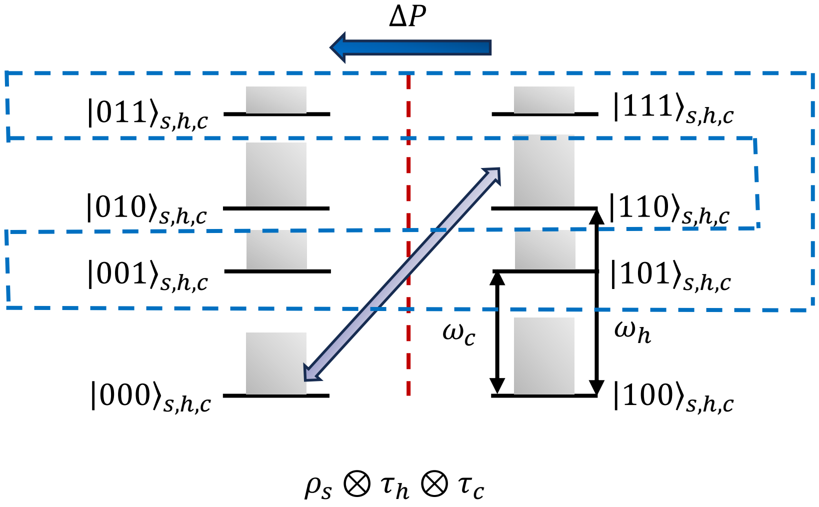

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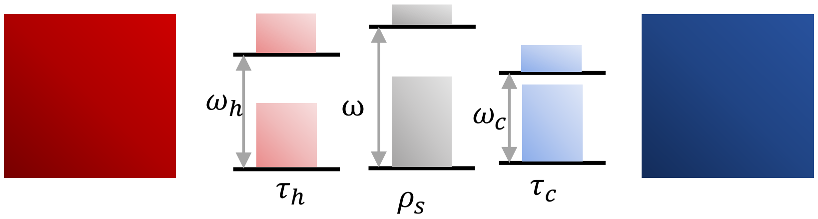
Catalytic enhancement



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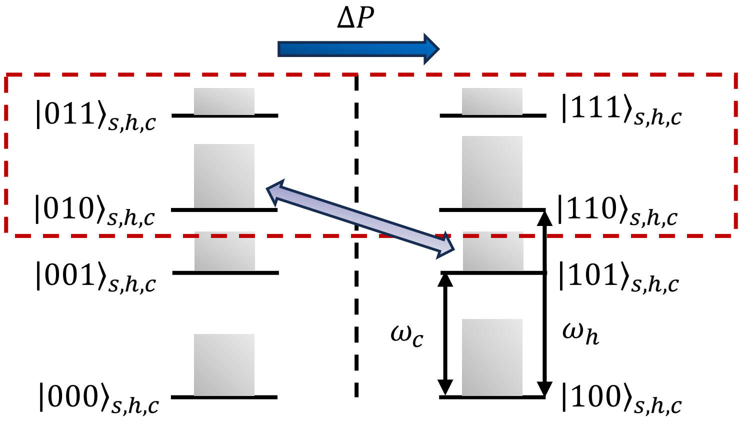
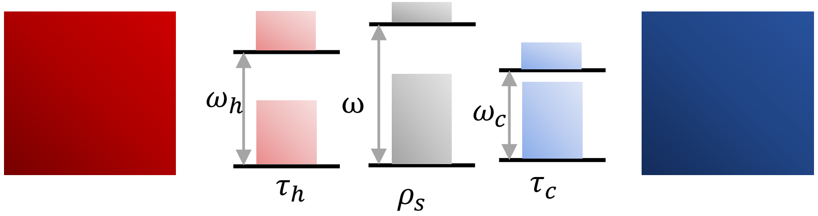


$\rho_s \otimes \tau_h \otimes \tau_c$

Catalytic enhancement



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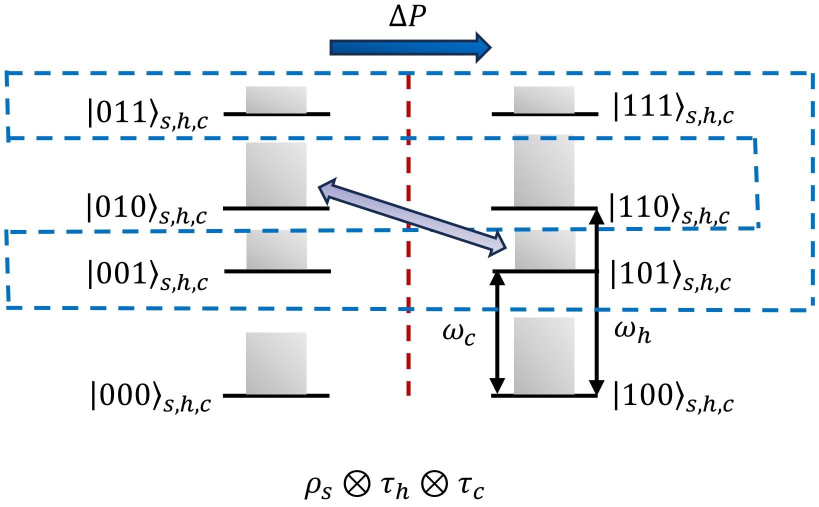

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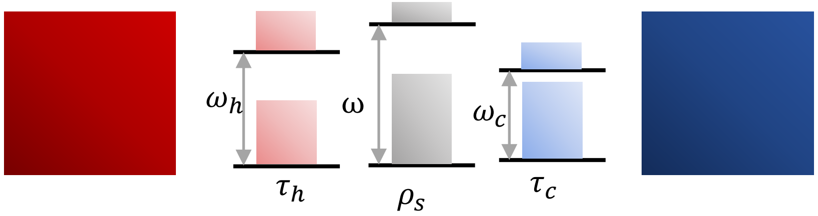

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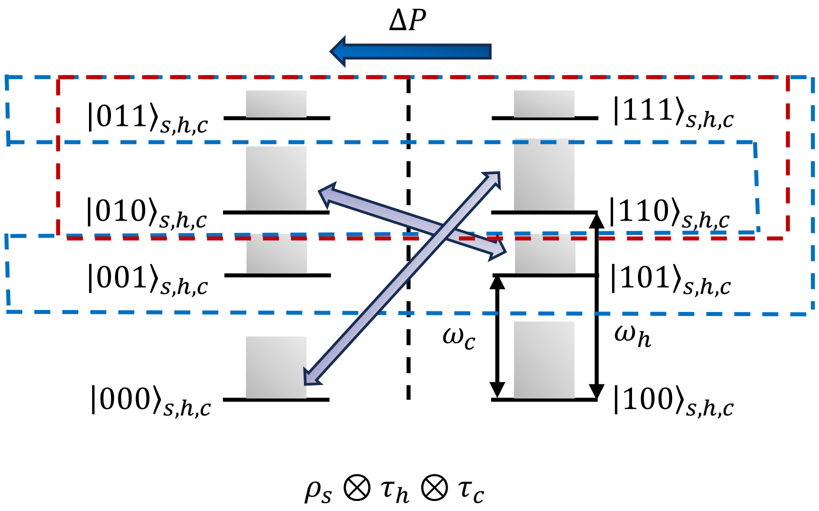

$$Q_h = Tr[H_h(\tau_h - \rho_h)] = 2\Delta P\omega_h$$

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Catalytic enhancement

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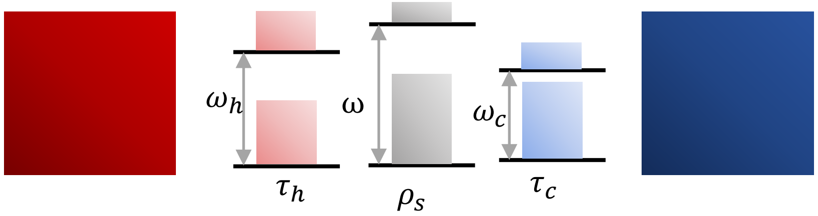

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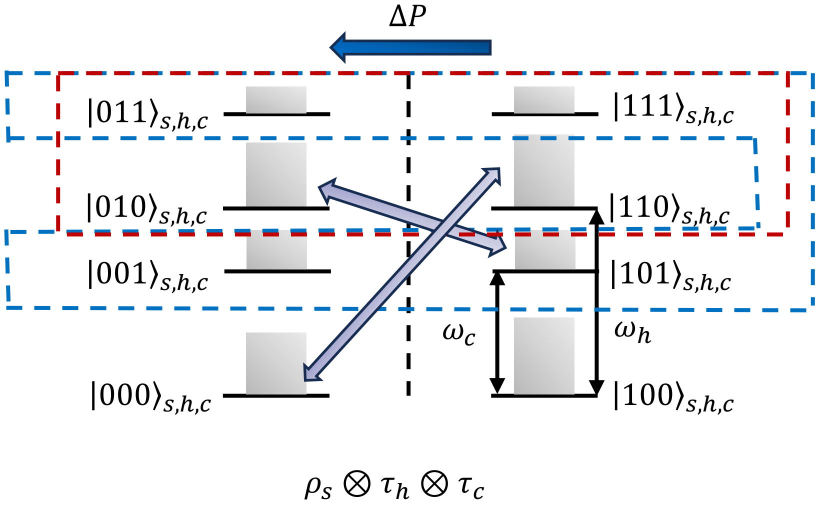
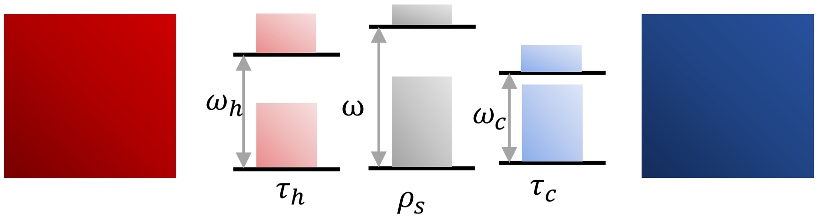


$Q_h = Tr[H_h(\tau_h - \rho_h)] = 2\Delta P\omega_h$

$Q_c = Tr[H_c(\tau_c - \rho_c)] = -\Delta P\omega_c$

$$\eta = 1 + \frac{Q_c}{Q_h} = 1 - \frac{\omega_c}{2\omega_h}$$

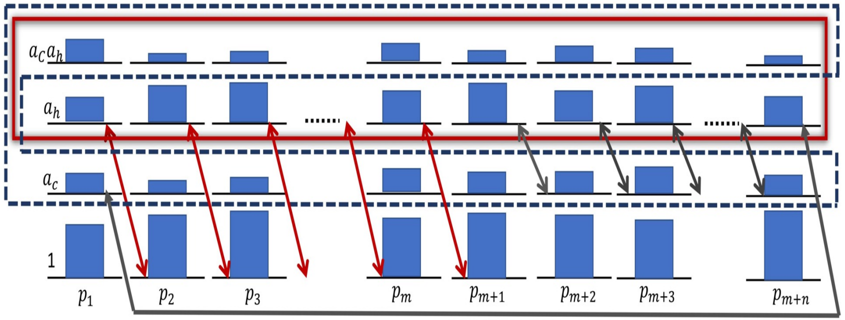
Catalytic enhancement



For a fixed value of ω_h and ω_c

Efficiency: $$\eta_2 = 1 - \frac{\omega_c}{2\omega_h} > 1 - \frac{\omega_c}{\omega_h}$$

$= \text{Optimal efficiency without catalyst}$

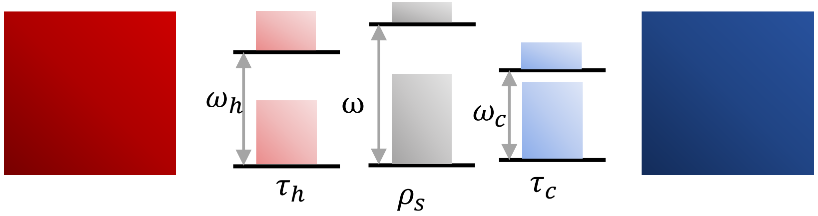


$\rho_s \otimes \tau_h \otimes \tau_c$

For a d-dimensional catalyst : $\eta_d = 1 - \frac{n\omega_c}{d\omega_h}$ $\quad n \in \{1, \ldots, d\}$

Catalytic enhancement

For a fixed value of ω_h and ω_c



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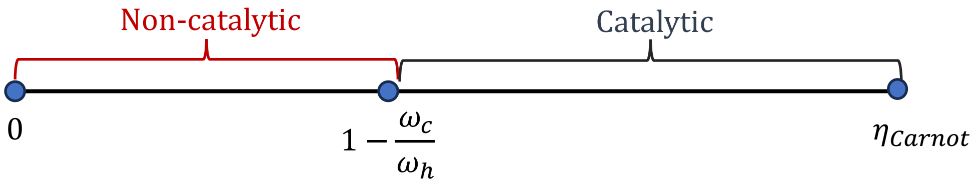
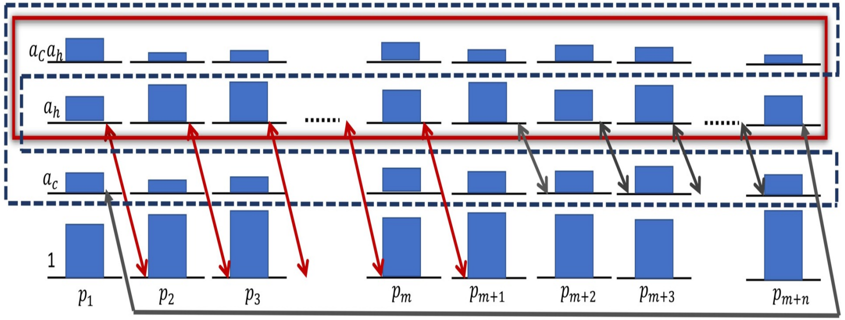


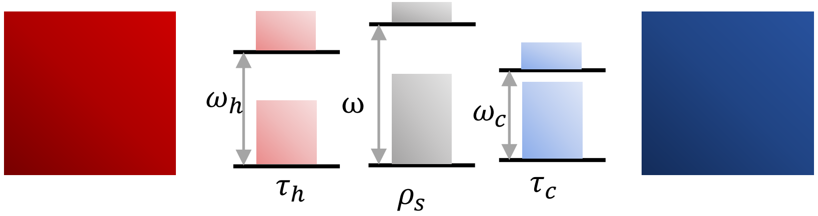


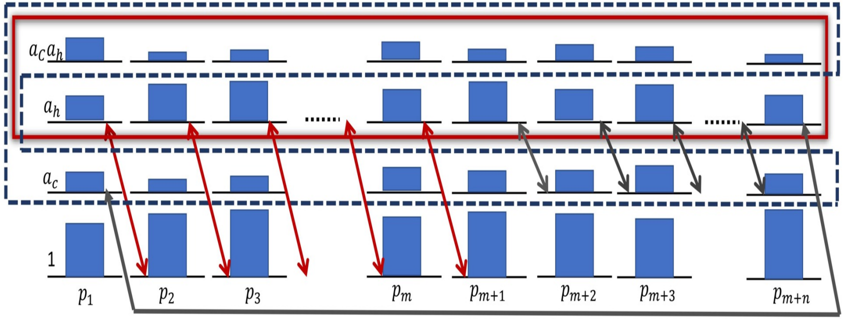
$\rho_s \otimes \tau_h \otimes \tau_c$

Catalytic enhancement

If $\omega_c > \omega_h$, then $1 - \frac{\omega_c}{\omega_h} < 0$;

But efficiency in catalytic scenario can still be positive depending on n and d : $\eta = 1 - \frac{n\omega_c}{d\omega_h} > 0$





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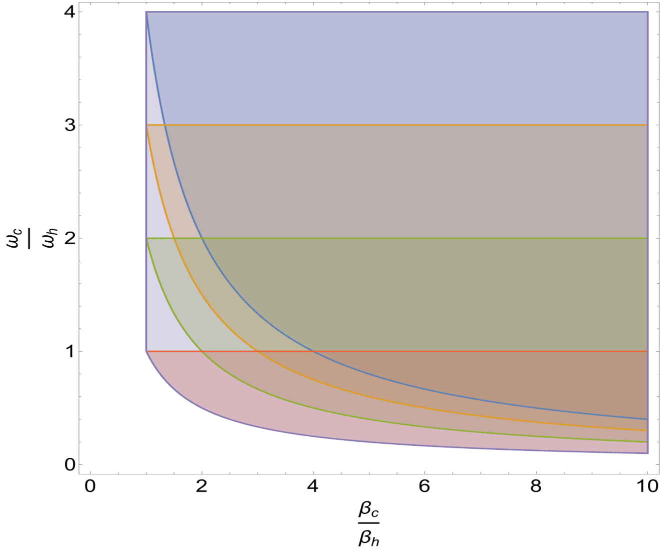
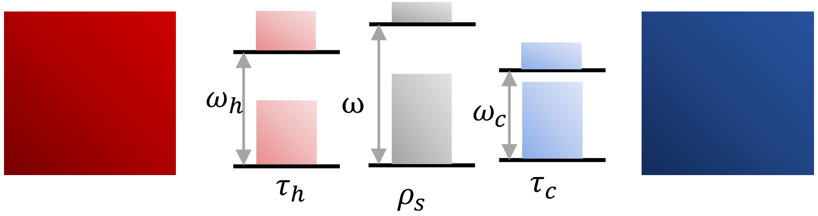
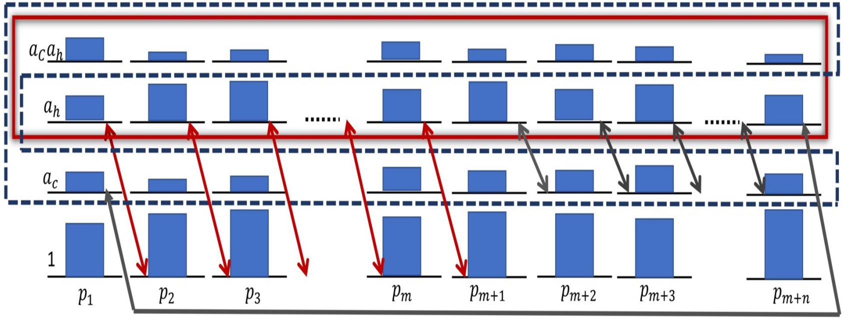
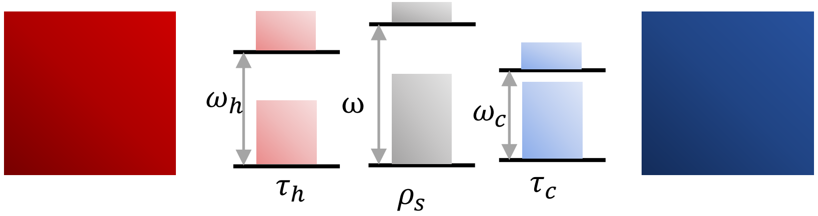




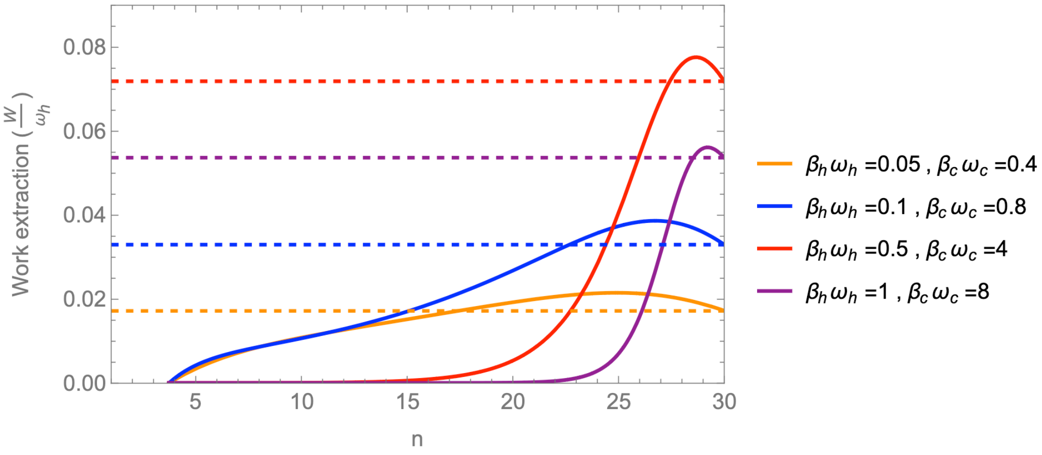


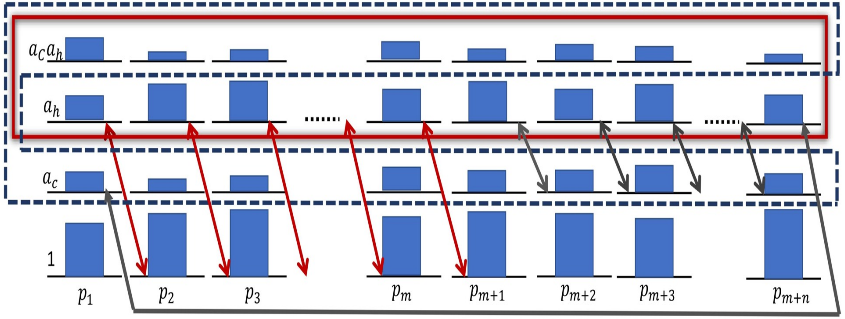
Catalytic enhancement

For a fixed value of ω_h and ω_c



$$W \propto \left(e^{-\beta_h \omega_h (m+n)} - e^{-n\beta_c \omega_c}\right) = \left(e^{-\beta_h \omega_h d} - e^{-n\beta_c \omega_c}\right)$$





Conclusion and Outlooks.

1) Describing the catalyst assisted two-stroke engine.

2) We have shown the efficiency and work per cycle can be enhanced by incorporating a catalyst.

3) Exploring the roles of catalysis in cooling of qubits.

4) Bridging the gap between the catalyst assisted stroke-based and continuous thermal machines.